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A HIGH FREQUENCY PHASE COMPARATOR FOR ROCKET INSTRUMENTATION

UNPUBLISHED PRELIMINARY DATA

OTS PRICE

by

J. P. Froehlich

June 10, 1964

IONOSPHERE RESEARCH LABORATORY

University Park, Pennsylvania

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Submitted by: John S Nisbet
J. S. Nisbet, Associate Professor of Electrical Engineering, Project Supervisor

Approved by: A. H. Waynick
A. H. Waynick, Director, I. R. L.

The Pennsylvania State University

College of Engineering

Department of Electrical Engineering

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ABSTRACT

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An analysis of a high frequency phase comparator using diode vector rectifiers is presented. The output voltage as a function of phase angle is determined. Errors arising from assuming a sinusoidal output are analyzed along with errors due to incorrect alignment of the phase splitting network used to obtain quadrature outputs. Effects of temperature on the components are investigated.

author

1. Introduction

1.1 Origin of the Problem

A high altitude rocket experiment was proposed by Nisbet, et al, (1961) to determine the absolute electron density by means of a separating rocket capsule technique. The system required the accurate phase measurement of two sub-harmonically derived frequencies with respect to a reference frequency. A phase detector capable of measuring phase angles within $\pm 5^{\circ}$, with the additional requirements of light weight, small volume, and low power consumption was needed for a successful experiment. A unit was designed by Alan J. Fisher of Spacecraft, Inc., Huntsville, Alabama, the company which is designing and building the equipment to be used in the high altitude rocket experiment known as the Mother-Daughter project. The circuit is simple in design, requires fewer components than here-to-fore devices used for phase detection, and abnormal operation of the unit can easily be detected.

1.2 Specific Statement of the Problem

1) To investigate methods of measuring the phase difference between two 72 mc/s signals and to examine the relative suitability of each for a rocket borne application.
2) To analyze the phase comparator designed by Spacecraft, Inc. in order to determine the theory of its operation; and particularly, to find an expression for the output voltage as a function of phase difference between the two input signals.

3) To find potential sources of error such as deviations from sinusoidal output voltage, and to study the resolution and sensitivity of the unit along with errors arising from noise sources.

4) To examine the operation of the phase comparator under conditions of extreme temperature variation to determine its reliability as a rocket-borne device.

1.3 Review of Literature

The phase difference between two periodic sinusoidal voltages of the same frequency is the time difference between two identical points of each divided by the reciprocal of the frequency. The points usually taken are the points where the waveform crosses the time axis while moving in a positive direction.

A number of instruments have been developed to measure phase directly for frequencies between the U. H. F. band and sub-audio frequencies.

In the proposed rocket experiment the required phase measurement must be made in the capsule and it is advantageous to make the phase measurement of 72 mc/s. Therefore, phase comparators principally operating in this region will be investigated.

A number of papers on microwave phase comparators are included in the bibliography, however, as it is felt that it was worth investigating these techniques to determine possible relevance to the present problem. These papers include Schafer and Beatty (1960), Schafer (1960), Lacy (1961), Chasek (1962), Cohn (1962), Israelsen and Haegele (1962), Kaiser, Smith, Pepper and Little (1962), and Sparks (1962).

Y. P. Yu (1961) has classified phase measuring instruments in two general categories according to their operating principles. Included in the first category are instruments employing calibrated phase shifters and vector difference rectifiers. The rectifier uses a diode configuration which produces an output proportional to the vector difference of the two input voltages.

In this type of device a phase measurement is made by manually adjusting the phase shifters for minimum output from the vector difference rectifier. The phase difference is then obtained by reading the calibrated phase shifter.

This method of measuring phase can be quite accurate and is usable up to a frequency of 2000 mc/s. Below 400 mc/s an instrument of this type can measure phase to within about 1%.

To adapt this type of instrument for rocket experiments, a null detecting circuit would be needed to automatically maintain the minimum output of the vector difference rectifier and also to provide a voltage to a telemetry channel which would then relay the required information to the ground station. Since an accuracy of $\pm 5^{\circ}$ is all that is required for a successful experiment, the additional circuitry required could not be justified. Furthermore, it is unlikely that the original accuracy could be maintained with the inclusion of a null detecting device.

For devices in the second category, according to Yu, the input voltages are applied to limiter amplifiers, the outputs of which are then square-waves whose zero axis-intersections are preserved. The limiter maintains a constant amplitude regardless of signal variations. These two signals are then applied to a coincident generator which produces a square pulse with duration equal to the time interval between the leading edge of one square wave and the trailing edge of the other. A direct current meter calibrated in degrees then indicates the phase difference between the two inputs. The current which drives the meter could be used to provide a signal for a telemetry channel.

A phase detector of this type offers several advantages for rocket instrumentation. The output voltage is independent of input signal amplitude variations and an output signal which is a function of the phase difference between the inputs is readily available to drive a telemetry channel. Several instruments which use this method of operation are described by Alsberg and Lad (1949), Elliot (1962), Kretzman (1959), Woodbury (1961), and Yu (1951), (1953), (1958).

Two serious disadvantages of this particular type of phase comparator, so far as rocket-borne use is concerned, are the complex circuitry required and the additional space and power that is needed.

Another problem of a more serious nature is that the measurement of phase becomes unreliable at high frequencies due to the rise time of the output square waves and transients in the coincident generator.

Stevens (1960) developed a precision phasemeter employing a technique similar to that of the second category that can be designed to operate in a range from 20 mc/s to 520 mc/s with an accuracy of 2%. The method is complex, involving frequency division and pulse counting to obtain the required phase information, and is not practical for rocket instrumentation.

A third type of phase measuring device which uses the vector rectifier should be included. The two input voltages between which the phase difference is to be determined are applied directly to a vector difference rectifier and/or a vector summing rectifier. The resultant voltage is a direct indication of the phase angle.

Detwiler (1952) has analyzed two general types of phase comparators employing this principle of operation which he classifies into the "single-filter" and "double-filter" units depending upon whether the sum or difference voltages are combined in separate filters or a common filter. The characteristics of the "single-filter" detector are similar to the "double-filter" circuit except in the region where both input amplitudes are comparable. In this region the resolution of the "single-filter" detector is poor, which makes the unit undesirable as a phase sensing instrument.

In both the "single-filter" and the "double-filter" units the output is proportional to $E_s \cos \phi$, where φ is the phase angle between two input voltages and E_s is the amplitude of the smaller input, providing the ratio of the two input voltages is large. In the case of the "double-filter" detector, if the two inputs are equal in amplitude, the unit is usable as a phase sensing device and the output potential is nearly a linear function of the phase difference over the range 0 to 2π .

Detectors employing these techniques retain the advantage that they can be operated at high frequencies. There is, however, a loss in accuracy. If the ratio of E_1 to E_2 is kept large and E_2 , the smaller potential, is controlled in some manner to minimize fluctuations in amplitude, measurements can easily be made to within $\pm 5^\circ$. Because it meets the required tolerances, a phase comparator of this type is usable for rocket instrumentation.

In the case for which E_1 equals E_2 , achieving the required accuracy is further complicated because additional circuitry is required to control the amplitudes of both of the applied signals. This would cause undue complications for rocket instrumentation.

A disadvantage of a comparator whose output is proportional to a cosine function is the loss of phase sensitivity in the region of 0°. However, if a second comparator is added and a 90° phase shift is introduced in one of the voltages driving the second unit, one comparator is always operating in a region of maximum sensitivity with the least sensitivity of the combination occurring at 45°. The addition of the second comparator also eliminates the ambiguity of the direction of phase change.

A shunt modulator can also be used as a phase sensing device with an output which is proportional to the magnitude of the smaller input voltage and the cosine of the phase difference between the inputs under the condition that the ratio of the two input amplitudes is large. Fraser (1959) discusses such comparators and he gives an analysis of the error due to finite switching. He examines the case for which E_1 approaches E_2 and defines a correction factor, cos α , which is a function of the ratio of input voltages and of the phase difference between the two voltages applied to the modulator. This circuit is less efficient and offers no advantages over the rectifier circuits.

The circuit designed by Fisher (1962) employs vector detection rectifiers and offers several advantages over the units described by Detwiler. Although the output follows the familiar E cos ϕ relation, quadrature output can be obtained with the addition of only one extra diode. The circuit requires only a few components and operates successfully in the 72 mc/s frequency range. Output voltages which are functions of phase angles are readily available to drive telemetry channels. It is this circuit that will be examined in detail.

2. Theory of Operation

2.1 Circuit Description

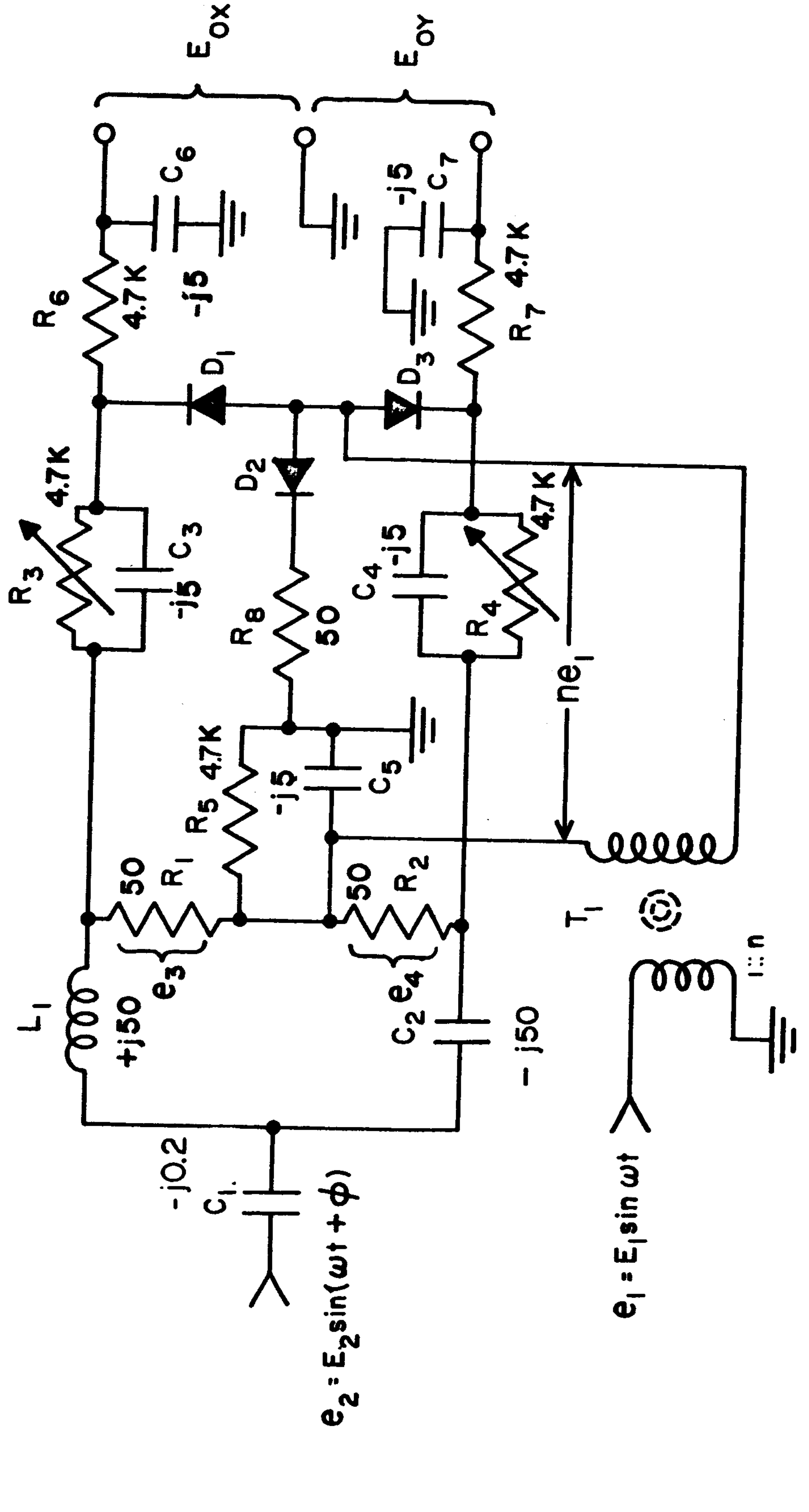
Figure 1 shows the complete phase comparator circuit. One input voltage driving the phase comparator is applied to a phase splitting network to obtain outputs dependent on both the sine and cosine of the input phase angle. This particular circuit develops the required phase shift for quadrature outputs by introducing a positive and negative 45° displacement in the input network.

To achieve the $\pm 45^\circ$ shift, the junction of R_1 and R_2 must be at a.c. ground. Other conditions for satisfactory operation of this unit require that this same junction present a high impedance to any d.c. potential present. This is accomplished by a parallel connection of R_5 and a large capacitor C_5. The coupling capacitor C_1 should be large to minimize any additional phase shift introduced by it. With this condition satisfied the capacitor is neglected in the analysis of the phase splitting network.

Two separate RL and RC branches are required, each producing leading and lagging 45° phase shifts in the output voltage with respect to e_2. This is achieved by making the inductive and the capacitive reactances in each branch equal to the resistive component in that branch.

The current in the inductive branch is

$$i_L = \frac{e_2}{R_1 + jX_L} \tag{2-1}$$



PHASE COMPARATOR CIRCUIT

FIGURE I

where X_L is the inductive reactance in ohms

and if $X_L = R_1$

$$i_L = \frac{e_2}{R_1 (1 + j)} \quad . \tag{2-2}$$

When transformed to polar form this gives

$$i_L = .707 \frac{e_2}{R_1} \angle - 45^\circ \quad . \tag{2-3}$$

A similar relationship exists for the capacitive branch and the current in this branch for equal capacitive reactance and resistive components is:

$$i_C = .707 \frac{e_2}{R_2} \angle + 45^\circ \quad . \tag{2-4}$$

Therefore, the voltage developed across the resistor in the inductive branch is

$$e_3 = .707\, e_2 \angle - 45^\circ \quad , \tag{2-5}$$

and the voltage across the resistor in the capacitive branch is

$$e_4 = .707\, e_2 \angle + 45^\circ \quad . \tag{2-6}$$

Thus the required phase shift is obtained at these junctions in the phase splitting network.

The input impedance is found from the total current flowing in the two branches.

$$i_2 = \frac{e_2}{R + jX_L} + \frac{e_2}{R - jX_C} \quad . \qquad (2\text{-}7)$$

Therefore,

$$Z_{in} = \frac{e_2}{i_2} = \frac{(R^2 + X_L X_C) + jR(X_L - X_C)}{2R + j(X_L - X_C)} \text{ ohms} \quad . \qquad (2\text{-}8)$$

where Z_{in} is the input impedance.

If X_L is chosen equal to X_C the circuit is antiresonant and a purely resistive impedance component is presented to the source voltage. Under these conditions

$$Z_{in} = R \qquad (2\text{-}9)$$

Since this circuit is designed to have a 50 ohm input impedance an inductive reactance of +j50 ohms along with a capacitive reactance of -j50 ohms is required to achieve proper operation.

The resistors R_3, R_4, and R_5 are used to balance each d. c. output to zero average level to compensate for possible mismatched properties of the three diodes. The values of these resistances should be small compared to the back impedance of the diodes or the efficiency of the phase comparator will be reduced. These resistances must be bypassed by the capacitors C_3, C_4, and C_5 to provide a low impedance path for the a. c. signals.

The resistors R_6 and R_7 in conjunction with the capacitors C_6 and C_7 are low pass filters which limit the system bandwidth.

The reference signal e_1 is applied to the phase comparator circuit by means of the step up transformer T_1 which transforms the low input impedance from the 50 ohm source and increases the voltage. The Q factor of the transformer when loaded by the non-linear diodes should be high to maintain a sine wave voltage drive to the three diodes.

The resistor R_8 is used to provide proper matching to the inputs e_3 and e_4 when the diodes are in a conducting condition.

The only requirements on the diodes are that they possess a high back to forward impedance ratio and a negligible effective capacitive susceptance at the frequency of operation. Low capacity silicon or germanium diodes satisfy these conditions. For operation at 72 mc/s, type 1N830A diodes have been found to be satisfactory.

To give a simple picture of the circuit operation, a limiting condition may be examined when ne_1 is very large compared with e_3 and e_4.

When ne_1 is negative diodes D_1, D_2, and D_3 are reverse biased and may be considered as open circuits. During this half of the cycle the voltage e_3 will charge C_6 through R_3 and R_6 in series and the voltage e_4 will charge C_7 through R_4 and R_7.

When ne_1 is positive then D_1, D_2 and D_3 may be considered to be forward biased and hence of low resistance. The voltages on C_6 and C_7 will discharge slowly through R_6 and R_7 respectively in series with R_8 to ground.

The d.c. voltages E_{ox} and E_{oy} will depend on the magnitude of e_3 and e_4 and on their phase relationship to ne_1.

The two halves of the circuit are independent because when the diodes are reversed biased, they are essentially unconnected and when the diodes are forward biased, their common terminal is connected to ground through a low resistance. For simplification, the analysis of each half of the comparator will be considered independently. Since the operation of each half is identical, the results will be combined in order to complete the analysis of the phase detection system.

2.2 Circuit Analysis

The equivalent circuit for each half of the phase comparator is shown in Figure 2. The potentials e_1 and e_3 may be written as

$$e_1 = E_1 \sin \omega t \tag{2-10}$$

and

$$e_3 = E_3 \sin (\omega t + \varphi) \tag{2-11}$$

where e_1 is the instantaneous a.c. potential at input 1

e_3 is the instantaneous a.c. potential at input 3

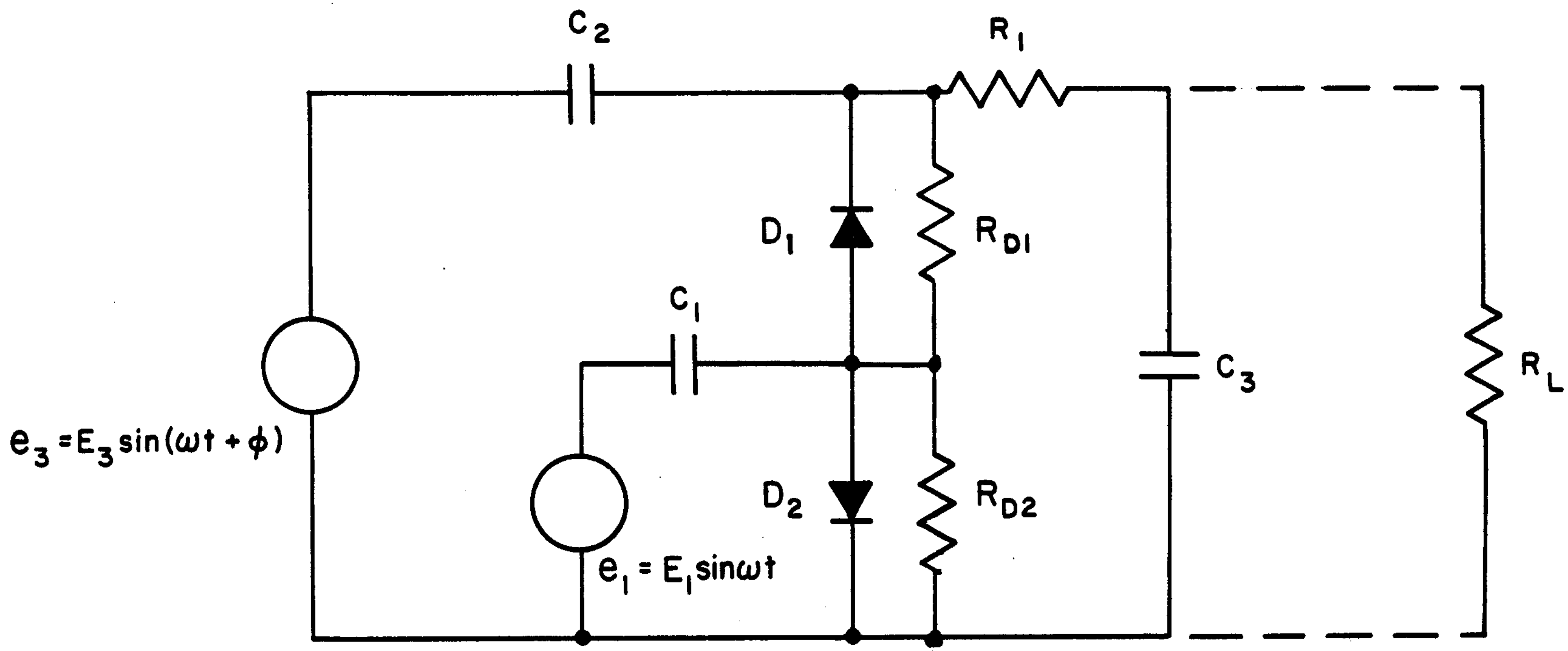
E_1 is the peak amplitude of the a.c. potential at input 1

E_3 is the peak amplitude of the a.c. potential at input 3

ωt is the angular frequency

φ is the phase difference between e_1 and e_3

The a.c. potential that appears across the top diode D_1 by Kirchhoff's law is the vector sum of e_1 and e_3 and is,



EQUIVALENT CIRCUIT FOR PHASE COMPARATOR

FIGURE 2

$$e_s = (E_1^2 + E_3^2 + 2\,E_1 E_3 \cos \varphi)^{1/2} \sin(\omega t + \delta) \qquad (2\text{-}12)$$

where δ is the displacement angle between e_1 and e_s, whereas the potential across the lower diode D_2 remains

$$e_1 = E_1 \sin \omega t \quad . \qquad (2\text{-}13)$$

At this point linear analysis no longer applies and the rectifying action of the diode must be considered. If a periodic signal is applied to an ideal diode, the diode will either present an infinite impedance or short circuit to the current depending upon the polarity of the voltage applied. Both a. c. and d. c. components of voltages present across the two diodes must be considered.

Terman (1947) discusses the diode voltmeter or vector rectifier. If R_{D_2}, the load impedance, is such that the efficiency of the rectification is high, the voltage developed across C_1 and hence R_{D_1} approaches the peak amplitude of the applied voltage E_1. The same process occurs across R_{D_1} but the peak amplitude in this case is $(E_1^2 + E_3^2 + 2\,E_1 E_2 \cos \varphi)^{1/2}$. R_{D_1} and R_{D_2} are resistances shunted across the diode. η, the efficiency of rectification, may be defined as

$$\eta = \frac{E_{d.c.}}{E_p} \qquad (2\text{-}14)$$

where $E_{d.c.}$ is the d. c. voltage across the diode

E_p is the peak value of the input voltage .

The efficiency becomes higher as the total load resistance is increased compared to the forward resistance of the diode. In this particular application, the load impedances R_{D_1} and R_{D_2} are infinite. In the ideal diode having a linear relationship between diode current and diode voltage, η is dependent only on the ratio of the load resistance to the diode resistance and is independent of the applied voltage. Practical diodes can be assumed to behave ideally if the applied signal is large compared to the knee in the V-I characteristics of the diode. The rectification efficiency decreases somewhat as the signal voltage becomes small and the diode impedance becomes dependent upon the applied voltage.

The output voltage developed across the capacitor C_3 will be the peak algebraic difference of the two applied voltages and is

$$E_o = \eta \left[E_1 - (E_1^2 + E_3^2 + 2 E_1 E_3 \cos \varphi)^{1/2} \right] \tag{2-15}$$

The a.c. components are eliminated by means of the low pass filters.

If $$E_1 >> E_3$$

$$E_o = \eta \left[E_1 - E_1 \left(1 + 2 \frac{E_3}{E_1} \cos \varphi \right)^{1/2} \right] \tag{2-16}$$

and because

$$\left(1 + 2 \frac{E_3}{E_1} \cos \varphi \right)^{1/2} \cong 1 + \frac{E_3}{E_1} \cos \varphi , \tag{2-17}$$

equation (2-16) reduces to

$$E_o = - \eta E_3 \cos \varphi . \tag{2-18}$$

Similarly if $E_3 >> E_1$

$$E_o = - \eta E_1 \cos \varphi \tag{2-19}$$

Thus, it can be seen that when $E_1 >> E_3$ or when $E_3 >> E_1$, the output voltage E_o is proportional to the negative of the cosine of the angle between the two inputs. The constant of proportionality is equal to the product of the smaller applied voltage and the efficiency of rectification.

If E_1 is equal to E_2, then,

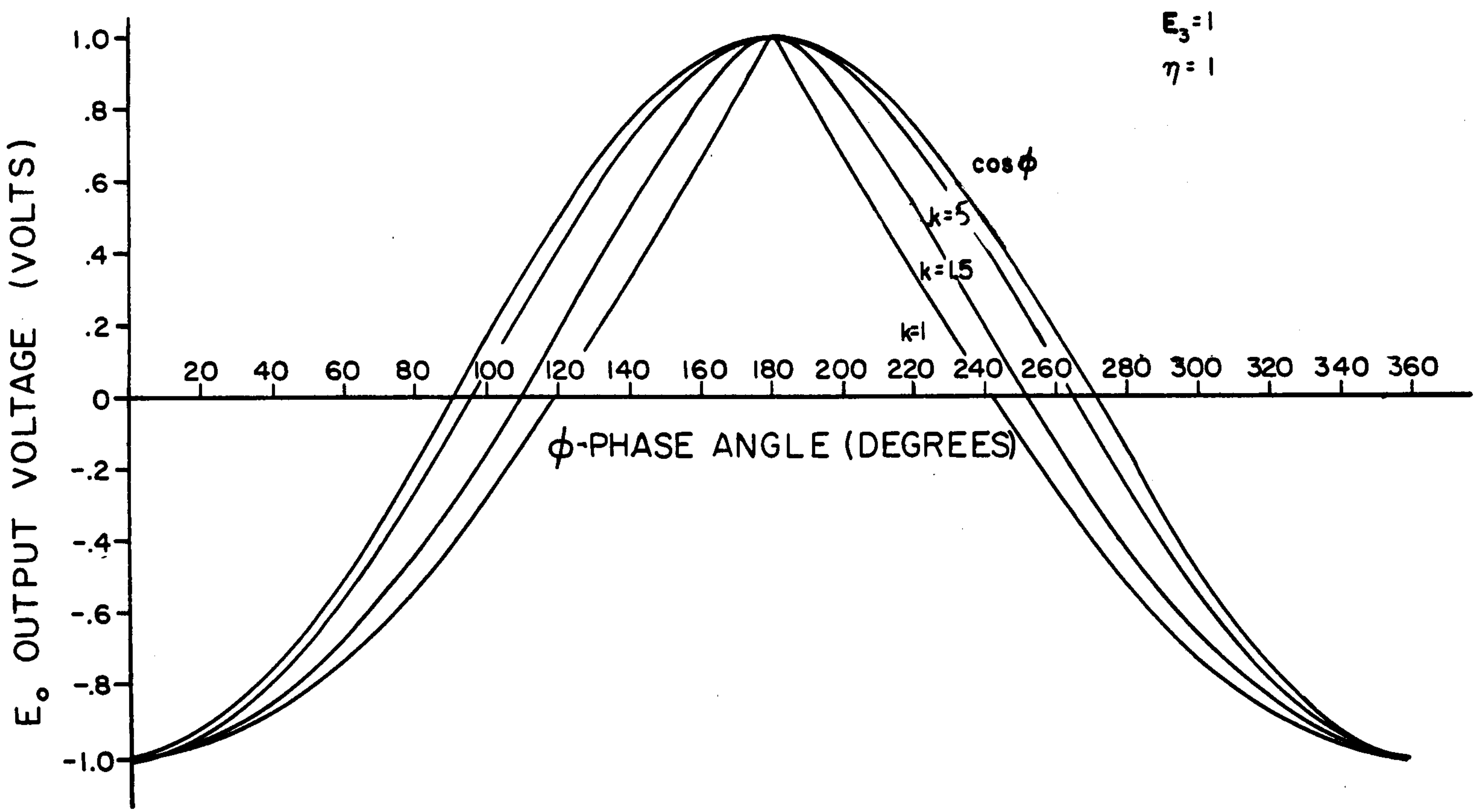
$$E_o = \eta [E_1 - E_1 (2 + 2 \cos \varphi)^{1/2}] \tag{2-20}$$

A plot of this output function assuming η equal to 1 is shown in Figure 3. It is apparent that symmetry no longer exists and that the output voltage is not a simple sinusoidal relationship.

To investigate the output voltage as a function of phase between E_1 and E_3 for several ratios of E_1 to E_3 the following definitions are made

$$k = E_1 / E_3 \tag{2-21}$$

For a large ratio k the output is proportional to the cosine of the angle between the two signals E_1 and E_3 . Clearly, from Figure 3 for the case k = 1, the cosine relationship no longer exists. The error voltage, ΔE, is defined as the difference between the output voltage E_o that is calculated for a given k from equation (3-16) and the output to be expected if a cosine function relationship existed.



OUTPUT VOLTAGE CHARACTERISTICS AS A FUNCTION OF THE PHASE ANGLE AND k

FIGURE 3

Therefore,

$$\Delta E = \eta E_3 \left[\cos \varphi + k - (k^2 + 1 + 2k \cos \varphi)^{1/2} \right] \qquad (2\text{-}22)$$

Figure 3 also shows the output voltage E_o as a function of phase for several values of k and Figure 4 shows the error voltage, ΔE, as a function of phase, ϕ, for the same values of k. As k becomes larger the error voltage decreases and the output voltage more nearly approximates a cosine function. To find the minimum ratio k to assure that the $\pm 5^{o}$ error limit is not exceeded, the error voltage function must be further investigated.

2.2.1 Rate of Change of Error Voltage with Respect to Phase

The first derivative of the error voltage with respect to ϕ assuming a constant k is

$$\frac{1}{\eta E_3} \frac{d(\Delta E)}{d \varphi} = \sin \varphi \left[1 - k(k \quad + 1 + 2k \cos \varphi)^{-1/2} \right] . \qquad (2\text{-}23)$$

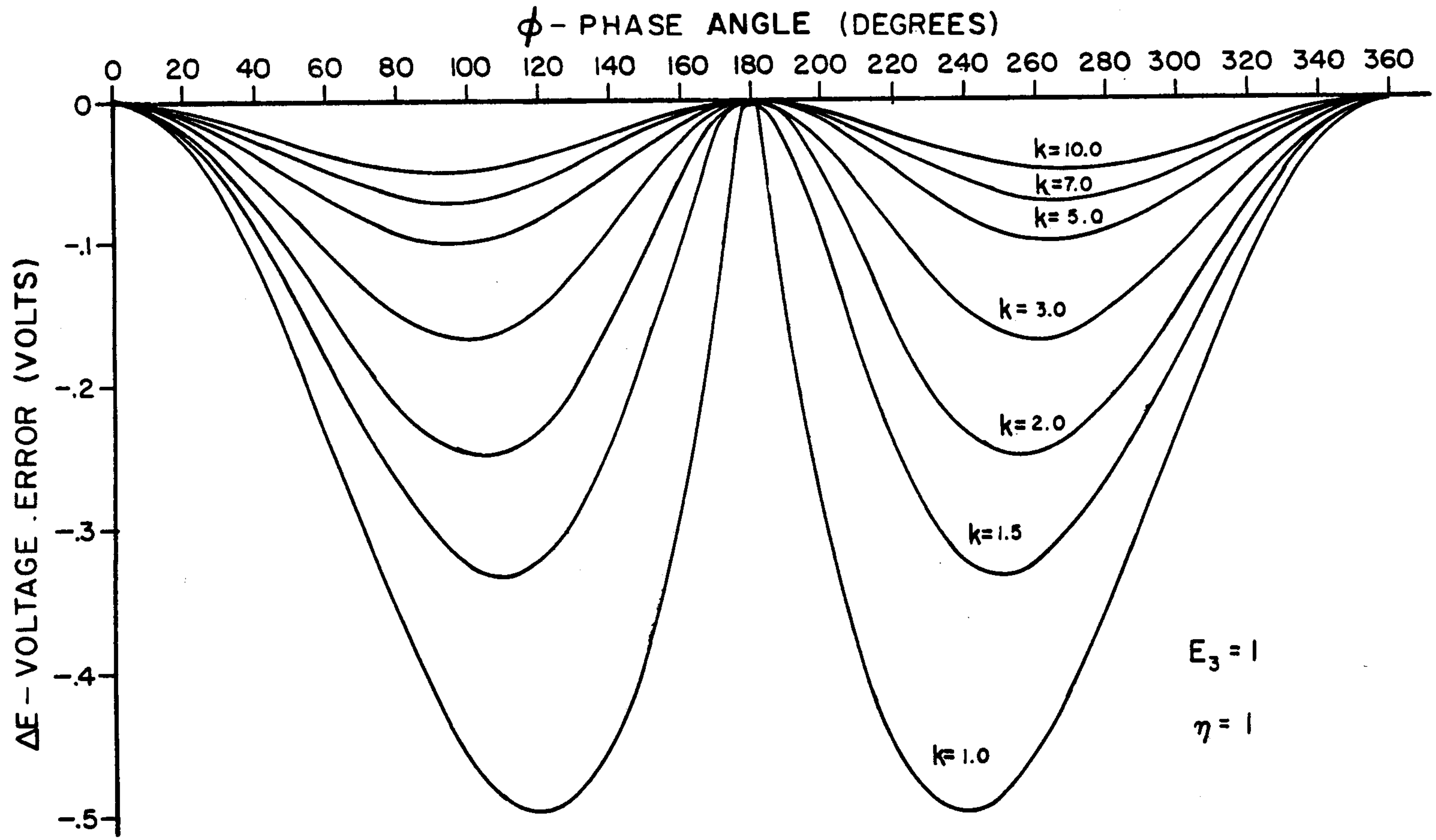
Critical points occur when $\frac{d(\Delta E)}{d \varphi} = 0$ and therefore when

$$\sin \varphi = 0 \qquad (2\text{-}24)$$

and

$$\cos \varphi = -1/2k . \qquad (2\text{-}25)$$

The second derivative of this function with respect to ϕ, maintaining k constant is,



VOLTAGE ERROR CHARACTERISTICS AS A FUNCTION OF PHASE ANGLE AND k

FIGURE 4

$$\frac{1}{\eta E_3} \frac{d^2(\Delta E)}{d\varphi^2} = \cos\varphi - k\cos\varphi\,(k^2 + 1 + 2k\cos\varphi)^{-1/2}$$

$$+ k^2 \sin^2\varphi\,(k^2 + 1 + 2k\cos\varphi)^{-3/2} \qquad (2\text{-}26)$$

When $\phi = 0^\circ$

$$\frac{1}{\eta E_3} \frac{d^2(\Delta E)}{d\varphi^2} = 1 - \frac{k}{k+1} \qquad (2\text{-}27)$$

Since k by definition is always positive and equal to or greater than 1, the second derivative for $\varphi = 0$, is always positive and the error is a minimum.

When $\varphi = 180^\circ$

$$\frac{1}{\eta E_3} \frac{d^2(\Delta E)}{d\varphi^2} = \frac{k}{k-1} - 1 \qquad (2\text{-}28)$$

Again the second derivative is always positive and the error is a minimum.

Since $\cos\varphi = -1/2\,k$ and k is always positive, this expression is valid between the limits of 90° and 120°. Solving in terms of this angle the following relationship is obtained

$$\frac{1}{\eta E_3} \frac{d^2(\Delta E)}{d\varphi^2} = \cos\varphi + \frac{1}{1 - 2\cos\varphi} + \frac{\sin^2\varphi\cos\varphi}{1 - 2\cos\varphi} \qquad (2\text{-}29)$$

$$90^\circ \leq \phi \leq 120^\circ$$

When $\varphi = 90^\circ$,

$$\frac{1}{\eta E_3} \frac{d^2 (\Delta E)}{d \varphi^2} = + 1 \quad . \tag{2-30}$$

If $\varphi = 120^{\circ}$, and $k = 1$,

$$\frac{1}{\eta E_3} \frac{d^2 (\Delta E)}{d \quad ^2} = - 0.335 \quad . \tag{2-31}$$

The theorem of concavity states that a graph of a smooth function is concave upward if the second derivative is positive and is concave downward if the second derivative is negative. Hence, the curve being considered has a point of inflection at a value of φ between 90° and 120°. The angle corresponding to this inflection point is 117.83° or 267.83°. The value of k for this angle is 1.0709.

Therefore, for a value of k less than 1.0709 the second derivative is negative and ΔE is a maximum. For k greater than 1.0709 the second derivative is positive and ΔE is relatively small.

2.2.2 Rate of Change of ΔE with Respect to k

When the angle φ is constant, the first derivative of ΔE with respect to k is

$$\frac{1}{\eta E_3} \frac{d (\Delta E)}{d k} = 1 - \frac{k + \cos \varphi}{(k^2 + 1 + 2 k \cos \varphi)^{1/2}} \tag{2-32}$$

Setting $\frac{d (\Delta E)}{d k} = 0$, it is found that the critical points occur at

$$\cos^2 \varphi = 1 \quad . \tag{2-33}$$

It is interesting to note that the critical points are independent of k and the error is always a minimum at the points where the phase angle is 0° and 180°.

2.3 Error as a Result of Resolution

The phase sensitivity is the derivative of the output voltage with respect to ϕ and is therefore a function of the sine of ϕ. This means that the minimum sensitivity occurs when the phase is 0° and 180°.

When E_o is taken to directly represent the phase angle, and no correction is added to adjust for the value of k, an additional error occurs. In Figure 5 graphs of the error in degrees are plotted as a function of phase angle for several values of k. This graph is similar to Figure 4.

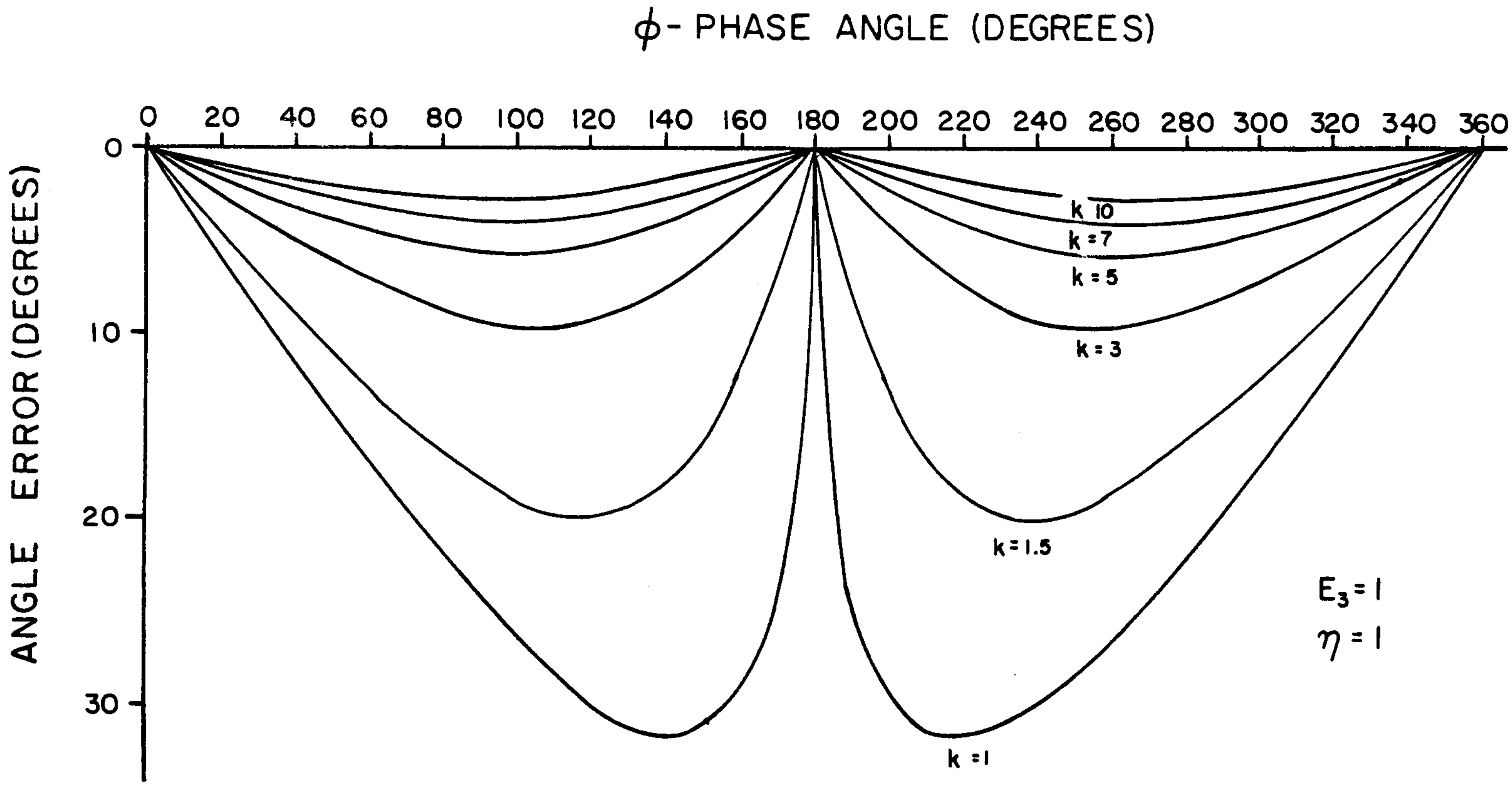
2.4 Determining the Value of k

If the value of k is known then the phase angle may be determined by using the relevant graph of ϕ vs. E_o for that k. An error arises if the value of k is not known and a sinusoidal output is assumed. The analysis shows that the maximum voltage error is always near 90° and 270° and that this error is minimized as k increases.

Figure 5 shows the magnitude of the error for an assumed sinusoidal relationship as a function of the phase angle ϕ. From Figure 5, it is apparent that for $k > 6$, the error is always within the $\pm 5^{\circ}$ limit.

2.5 Quadrature Output Phase Comparators

The phase comparator shown in Figure 1 gives output voltages



ANGLE ERROR CHARACTERISTICS AS A FUNCTION PHASE ANGLE AND k

FIGURE 5

which are functions of the sine and cosine of the phase angle between the two input signals. The network used to introduce the required phase shift between the input signals in order to provide quadrature outputs has already been described. As a result of this network, and the addition of a third, diode vector-rectifier, one channel is always passing through a region of high sensitivity. Another important feature of a comparator of this type is that the direction of phase change is always known.

2.6 Equation of Quadrature Output

If the signal from one channel of a phase comparator of the type just described is used to drive the horizontal deflection plates of an oscilloscope and the output from the other channel is used to drive the vertical plates, a Lissajous pattern is traced on the face of the scope. The shape of the pattern is determined by the value of k.

Under these conditions the voltage driving the horizontal plates is

$$E_{ox} = \eta E_3 \left[k - (k^2 + 1 + 2k \cos\varphi)^{1/2} \right] \tag{2-34}$$

and the voltage driving the vertical plates is

$$E_{oy} = \eta E_3 \left[k - (k^2 + 1 + 2k \sin\varphi)^{1/2} \right] . \tag{2-35}$$

Solving for cos φ and sin φ

$$\cos\varphi = \frac{\left(\dfrac{E_{ox}}{\eta E_3}\right)^2 - 2k\left(\dfrac{E_{ox}}{\eta E_3}\right)^2 - 1}{2k} \tag{2-36}$$

$$\sin\varphi = \frac{\left(\frac{E_{oy}}{\eta E_3}\right)^2 - 2k\left(\frac{E_{ox}}{\eta E_3}\right)^2 - 1}{2k} . \qquad (2\text{-}37)$$

Applying the trigonometric relationship

$$\sin^2\varphi + \cos^2\varphi = 1 , \qquad (2\text{-}38)$$

the equation of the pattern traced on the screen of the oscilloscope written implicitly in terms of E_{ox} and E_{oy} is·

$$\left[\left(\frac{E_{oy}}{\eta E_3}\right)^2 - 2k\left(\frac{E_{oy}}{\eta E_3}\right) - 1\right]^2 + \left[\left(\frac{E_{ox}}{\eta E_3}\right)^2 - 2k\left(\frac{E_{ox}}{\eta E_3}\right) - 1\right]^2 = 4k^2$$

(2-39)

This is the equation which describes the quadrature outputs, E_{ox} and E_{oy}, for the phase comparator.

For large values of k this expression reduces to

$$\left(\frac{E_{oy}}{\eta E_3}\right)^2 + \left(\frac{E_{ox}}{\eta E_3}\right)^2 = 1 \qquad (2\text{-}40)$$

which is the equation for a circle of unit radius.

Figures 6, 7, 8, 9, 10, and 11 are plots of E_{ox} and E_{oy} for values of k equal to 1, 1.5, 3, 5, 7, and 10, respectively. Even for k = 10, deviations from a circular trace are observed.

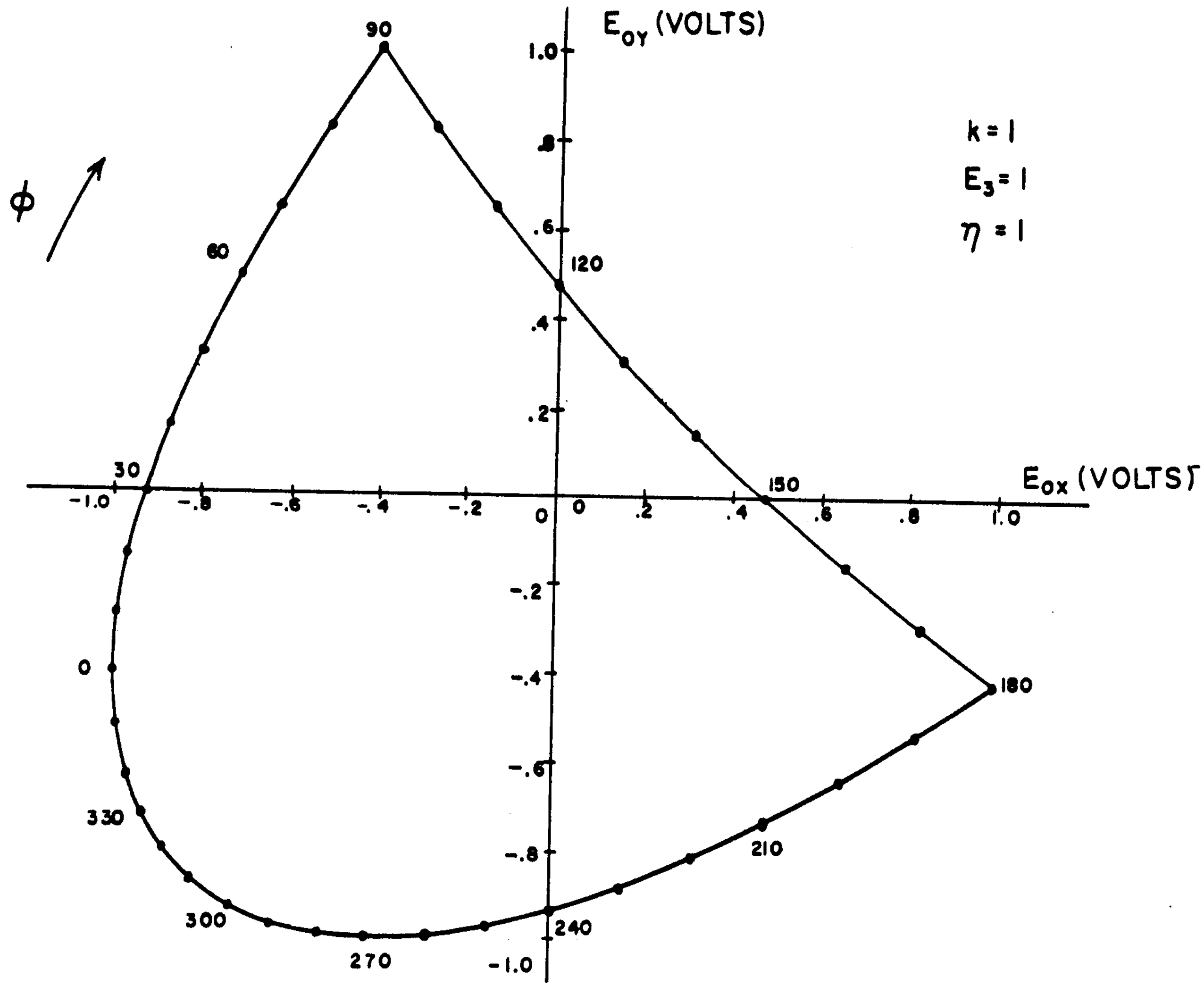


FIGURE 6 - QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR WITH PHASE ANGLE - k=1

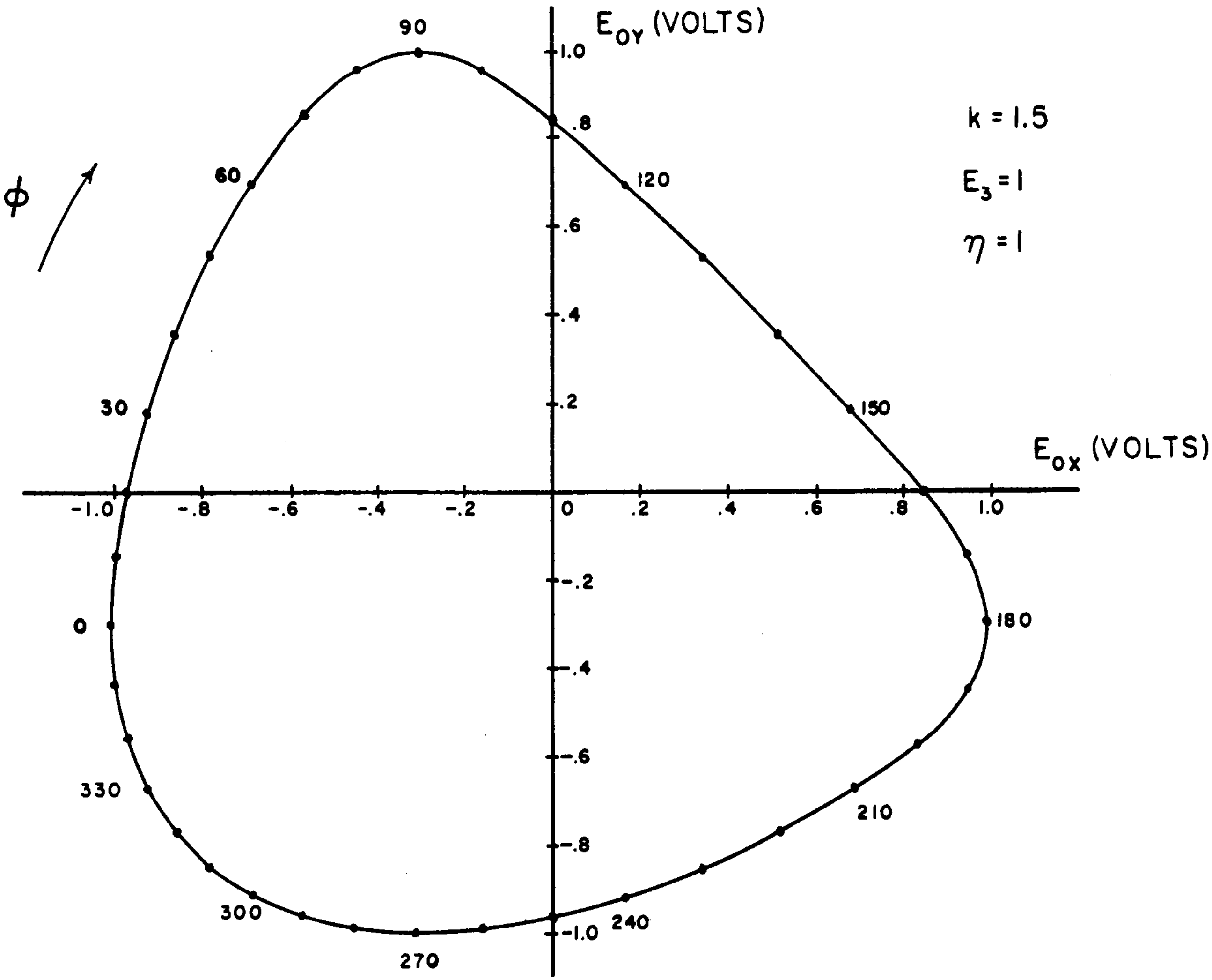


FIGURE 7 – QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR WITH PHASE ANGLE – k=1.5

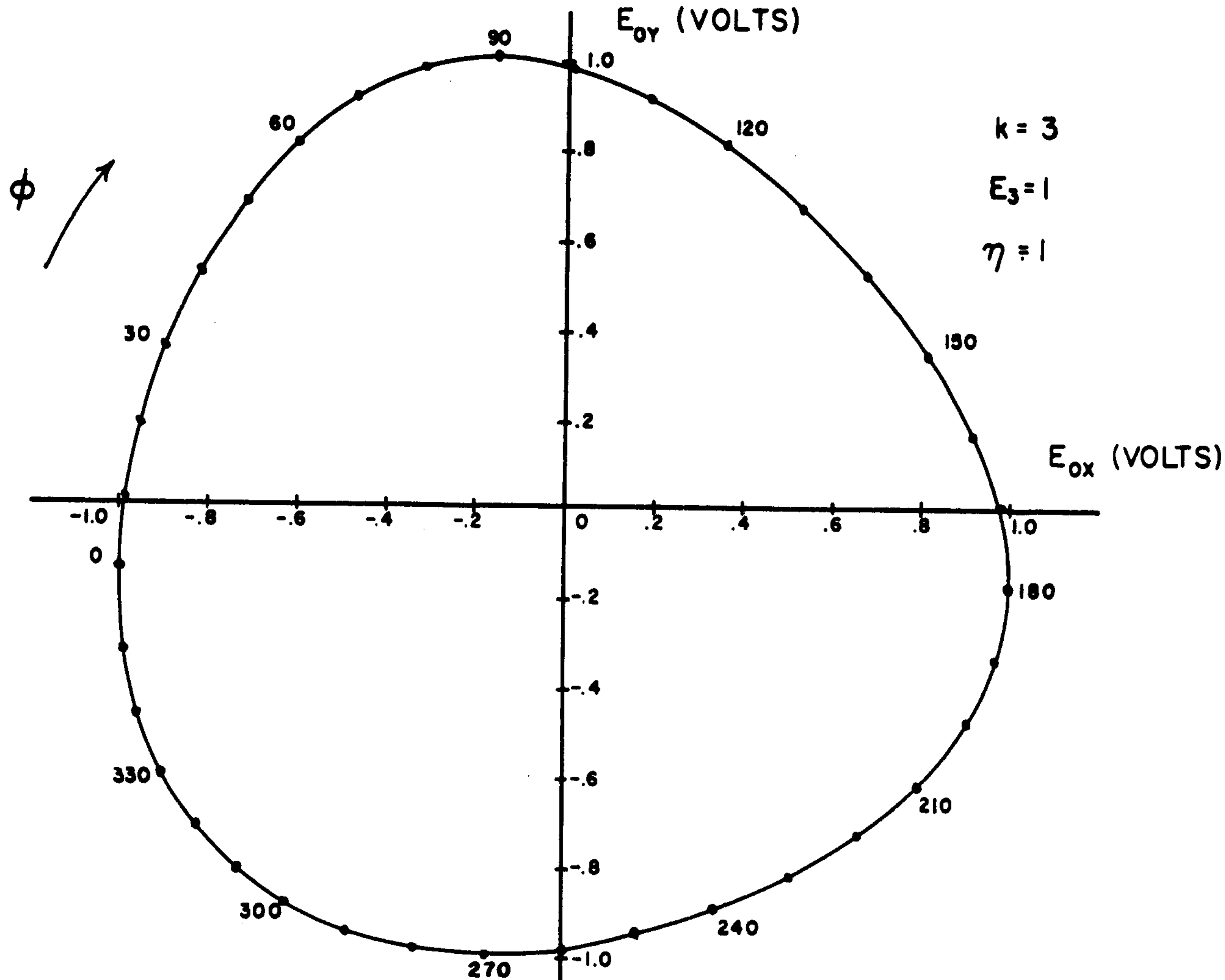


FIGURE 8 - QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR WITH PHASE ANGLE - k = 3

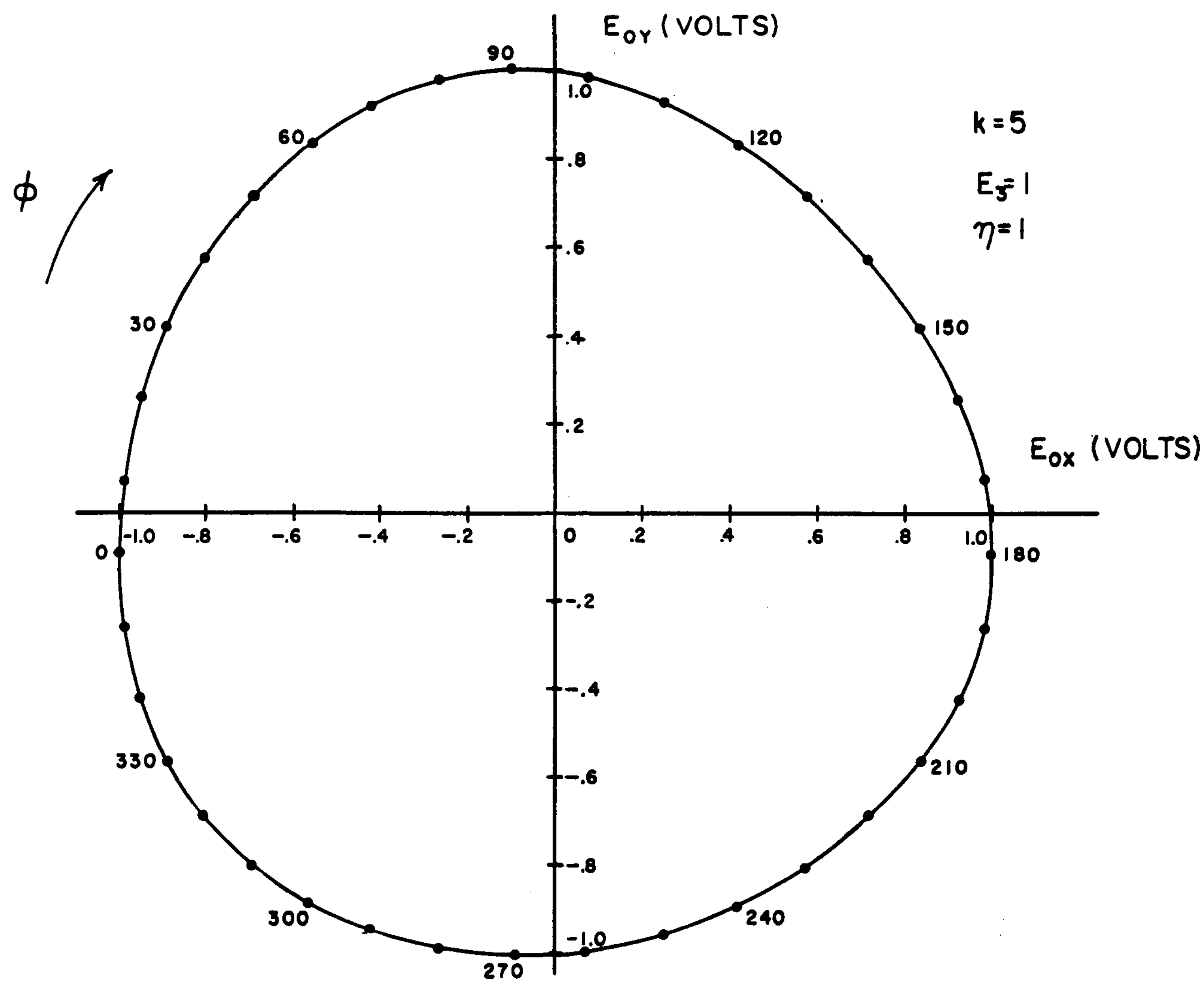


FIGURE 9 QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR WITH PHASE ANGLE k = 5

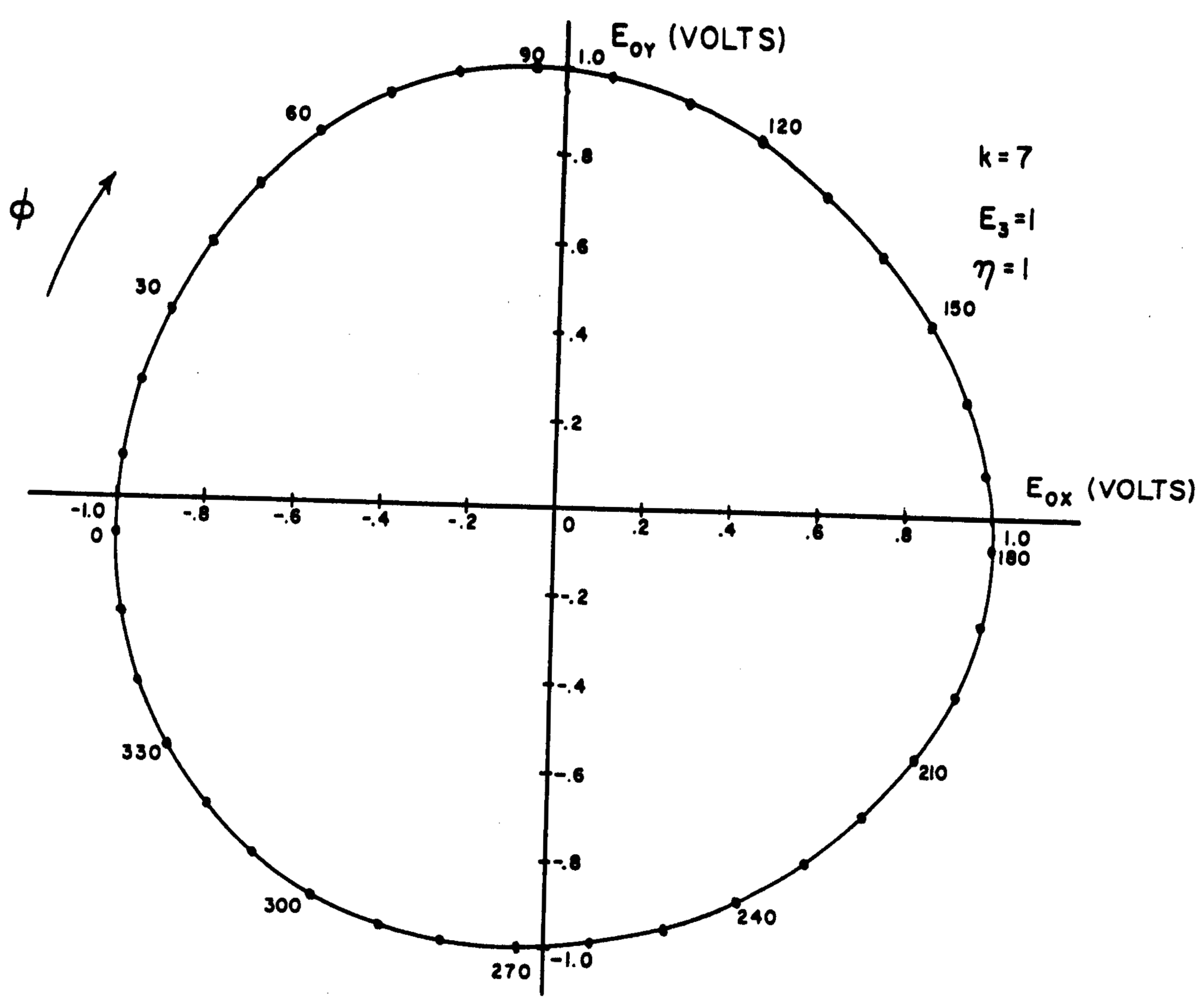


FIGURE 10 QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR WITH PHASE ANGLE k=7

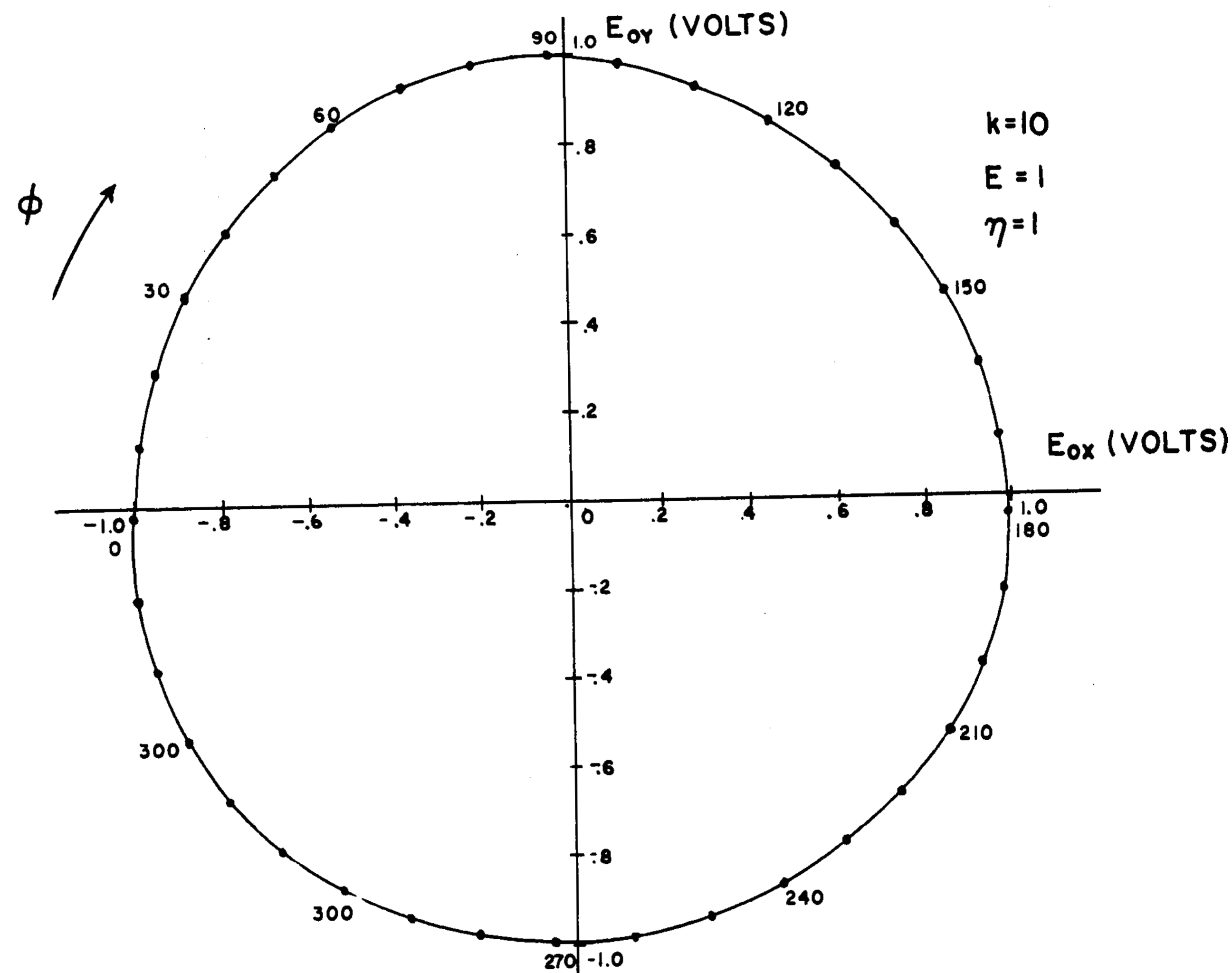


FIGURE 11 QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR WITH PHASE ANGLE k=10

3. Analysis of the Error in the Phase Comparator

3.1 Sources of Error

There are four main sources of error in the phase comparator and each source is treated independently. They are as follows:

1. Error arising from the value of k.
2. Error arising from incorrect alignment of the phase shift network.
3. Error arising from noise sources.
4. Error arising from temperature sensitive components.

3.1.1 Error Arising from the Value of k

Accurate phase measurement requires a precise knowledge of k. It is proposed that in the rocket experiment for which this phase comparator study was made, the value of k be determined from AGC voltages. The phase angle can then be determined by referring to the appropriate phase vs. voltage curve and thus no assumption about the functional form for phase vs. voltage will be necessary. However, if AGC information is not available, the value of k can be approximated from the amount of distortion in the phase comparator output voltage vs. time waveform. As long as k remains greater than 1 the maximum output voltage from each channel of the phase comparator is $\pm \eta E_3$ and occurs at phase differences of 0° and 180°. For k much greater than 1 the output will be sinusoidal. As k approaches 1, the output will become distorted with maximum distortion occurring for k equal to 1. For values of k approximately equal to 1 the maximum output, $\pm \eta E_3$, occurring at phase differences of 0° and 180° will decrease, with its amplitude then dependent upon the smaller of the two input voltages.

The proposed experiment requires a phase measurement to within $\pm 5^{\circ}$. The phase comparator described in this report will provide the required resolution providing the ratio of the input voltages remain larger than 6.

3.1.2 Error Arising from Incorrect Alignment of the Phase Shift Network

The purpose of the quadrature output is to obtain high sensitivity over the entire range of measurement and to resolve the ambiguity of the direction of phase change. So long as the phase displacement between the reference signals for each channel is 90°, phase quadrature exists between the outputs and minimum sensitivity occurs at 45°. Any deviation from this 90° phase displacement affects the sensitivity and thus reduces the resolution of the unit. The worst case would of course be with no phase shift between the two references which would result in the two outputs being in phase rather than in phase quadrature.

Since both channels are on the same time base the alignment of the phase shift network can be checked by observing the maxima of each channel. For correct alignment the maximum of one channel is positioned equidistant between two adjacent maxima of the other channel. Displacements to either the right or left of this position indicate a lag or lead in the phase splitting network.

A common method used in data analysis of quadrature phase comparators is to take the ratio of the outputs as a measure of the tangent of the phase angle. The phase angle is then given by:

$$\phi = \tan^{-1} \frac{\eta E_2 \sin \varphi}{\eta E_2 \cos \varphi} \tag{3-1}$$

The advantage of obtaining the phase angle ϕ by this method is that the measurement is independent of the efficiency and the magnitude of the smaller input voltage.

For the phase comparator considered here, the following equation for φ is obtained,

$$\varphi = \tan^{-1} \left[\frac{k - (k^2 + 1 + 2\,k \sin \varphi)^{1/2}}{k - (k^2 + 1 + 2\,k \cos \varphi)^{1/2}} \right] \tag{3-2}$$

For k much greater than 1 this reduces to the form of equation (3-1).

The error, δ , is defined as the difference between the actual phase angle and the angle given by equation (3-2). A plot of this error is given in Figure 12 for several values of k.

Figures 13, 14, 15, 16, and 17 show the effect of incorrect alignment of the phase shifter network when equation (3-2) is used to determine the phase angle φ . Increments of 5^{o}, 10^{o}, 15^{o}, 30^{o}, and 45^{o} of phase departure from quadrature are shown for the same values of k. The analysis shows that considerable error may be introduced because of incorrect alignment of the phase shifters if a division

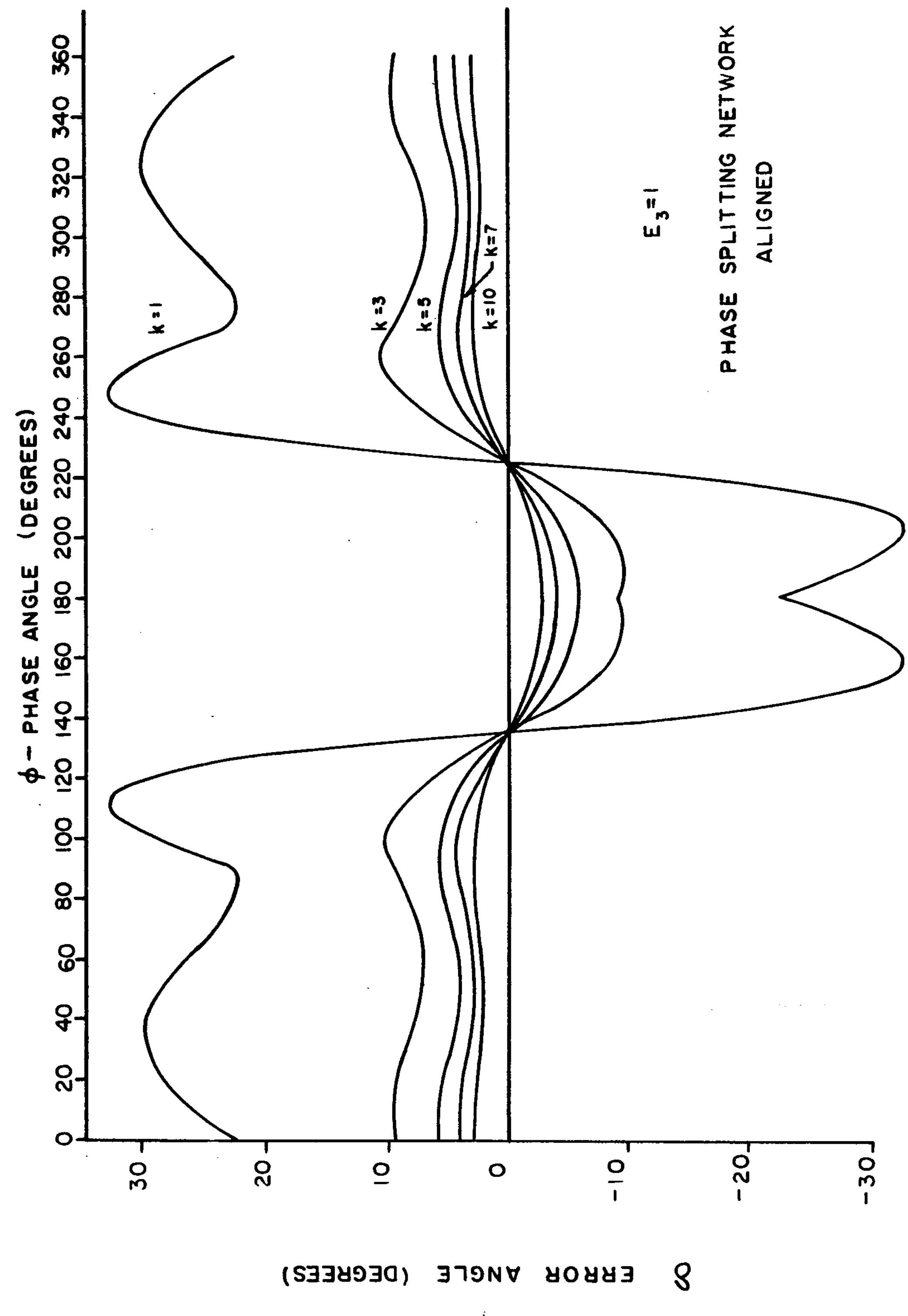
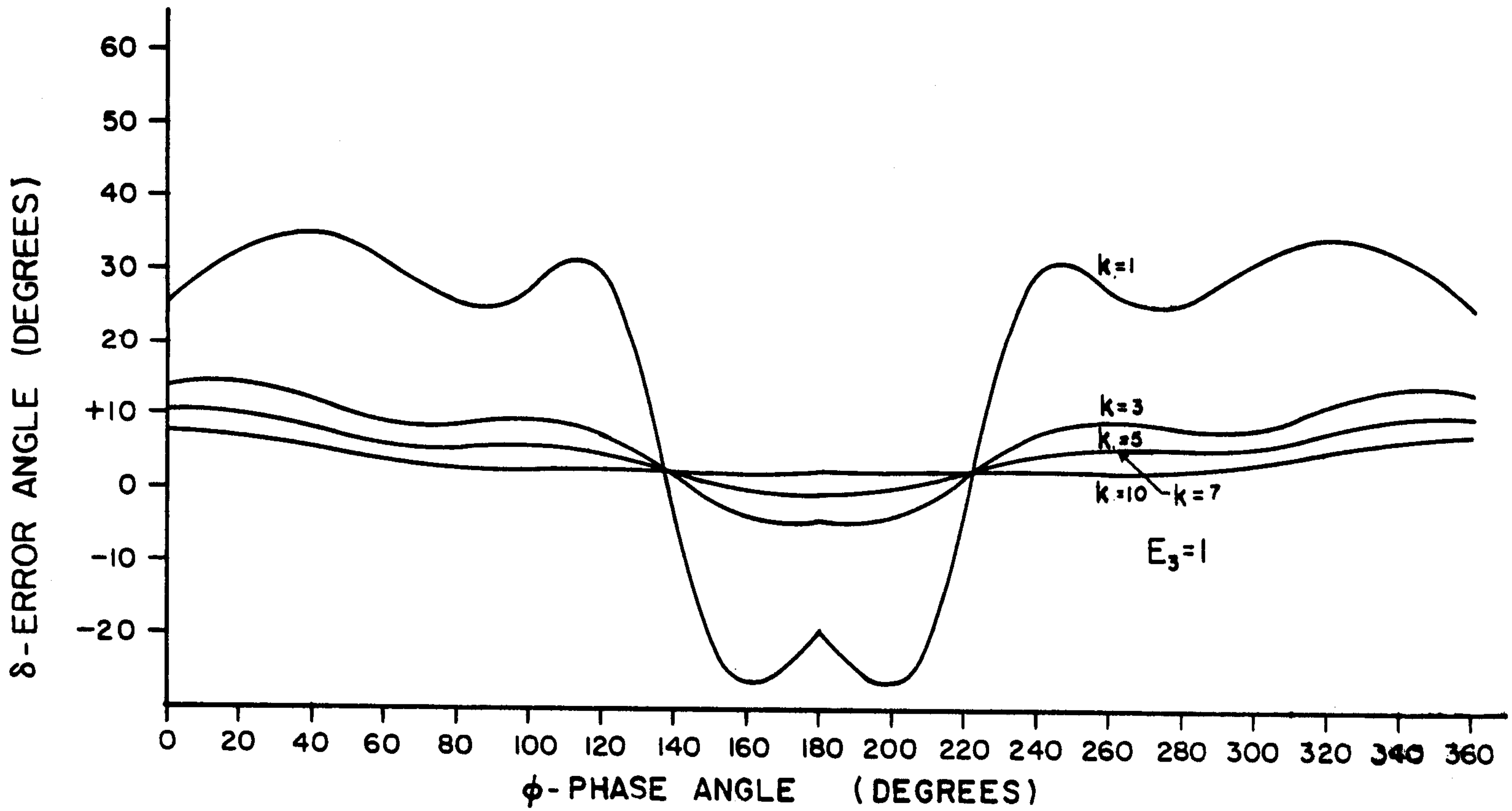


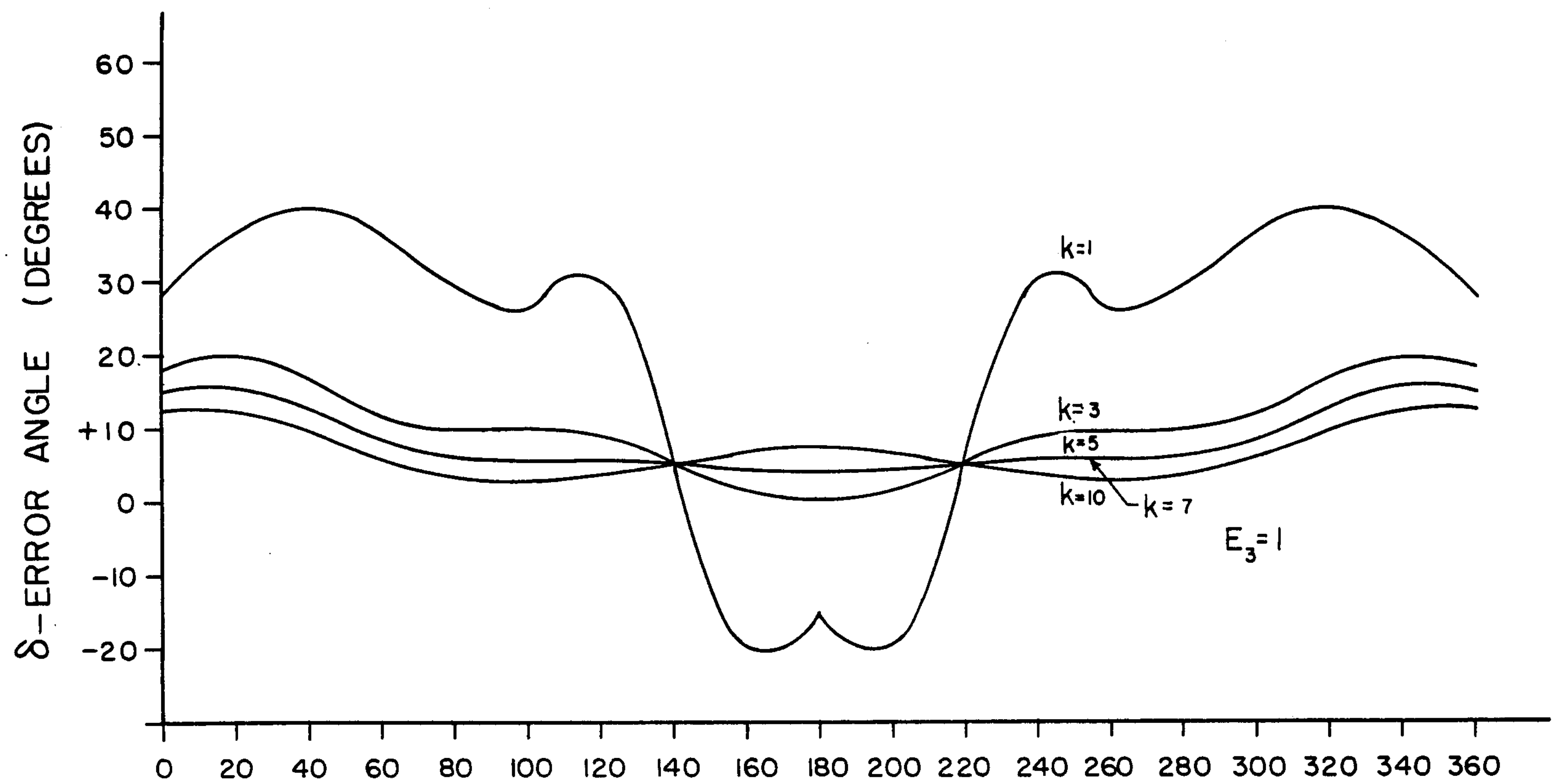
FIGURE 12 ERROR ANGLE Vs. PHASE ANGLE



ERROR ANGLE VS PHASE ANGLE

PHASE SPLITTING NETWORK INCORRECTLY ALIGNED 5°

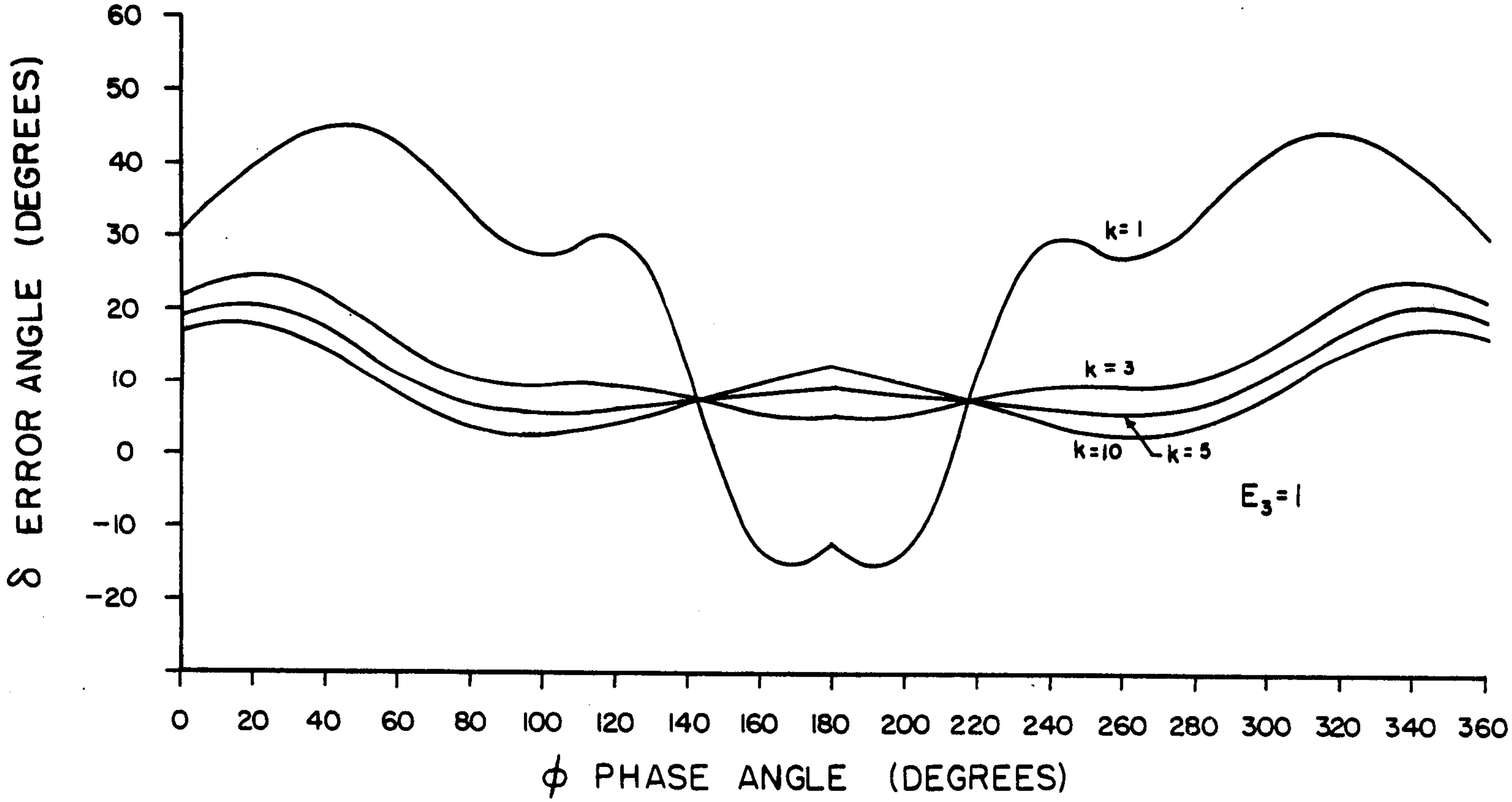
FIGURE 13



ERROR ANGLE VS PHASE ANGLE

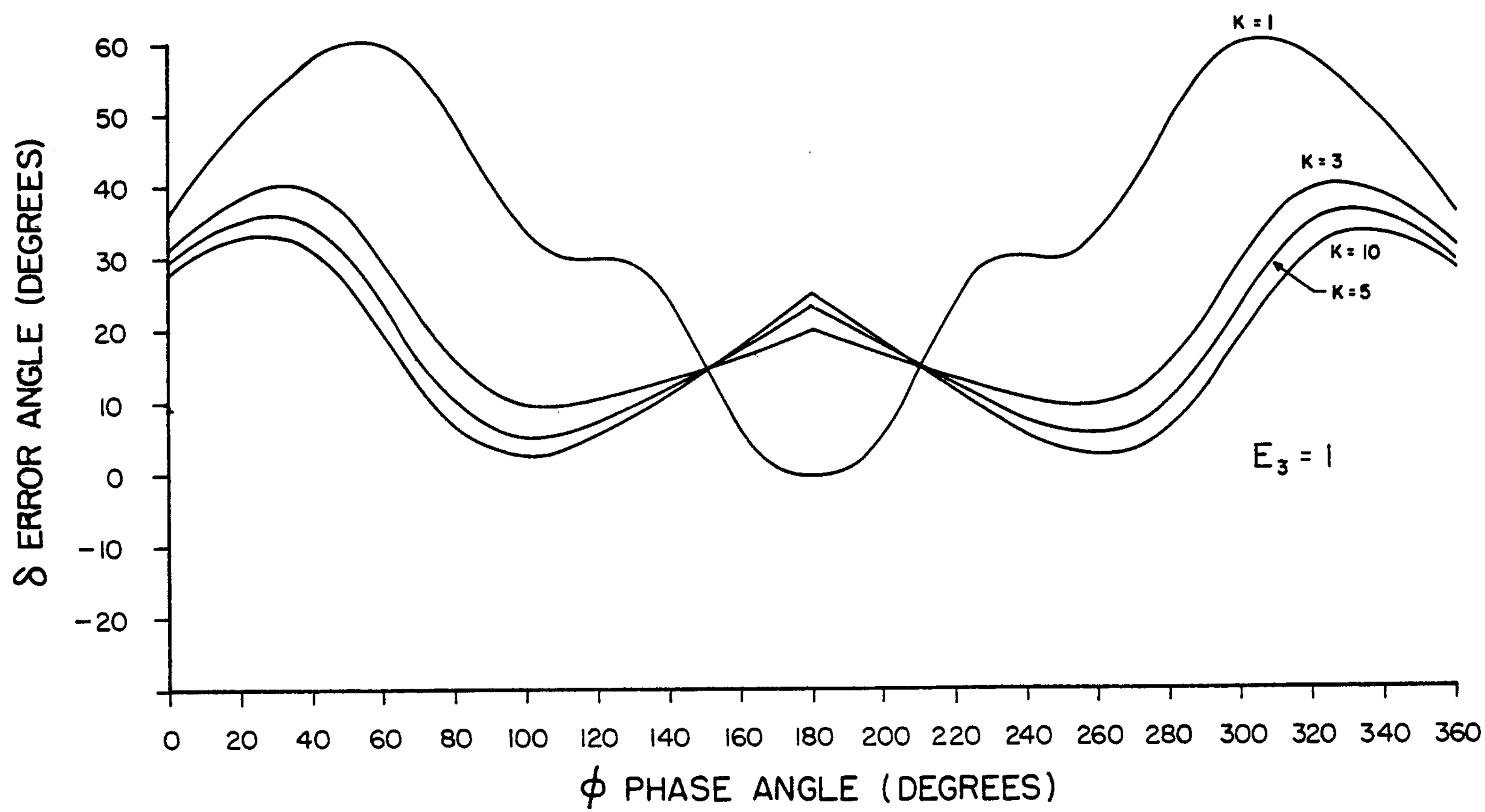
PHASE SPLITTING NETWORK INCORRECTLY ALIGNED 10°

FIGURE 14



ERROR ANGLE VS. PHASE ANGLE
PHASE SPLITTING NETWORK INCORRECTLY ALIGNED 15°

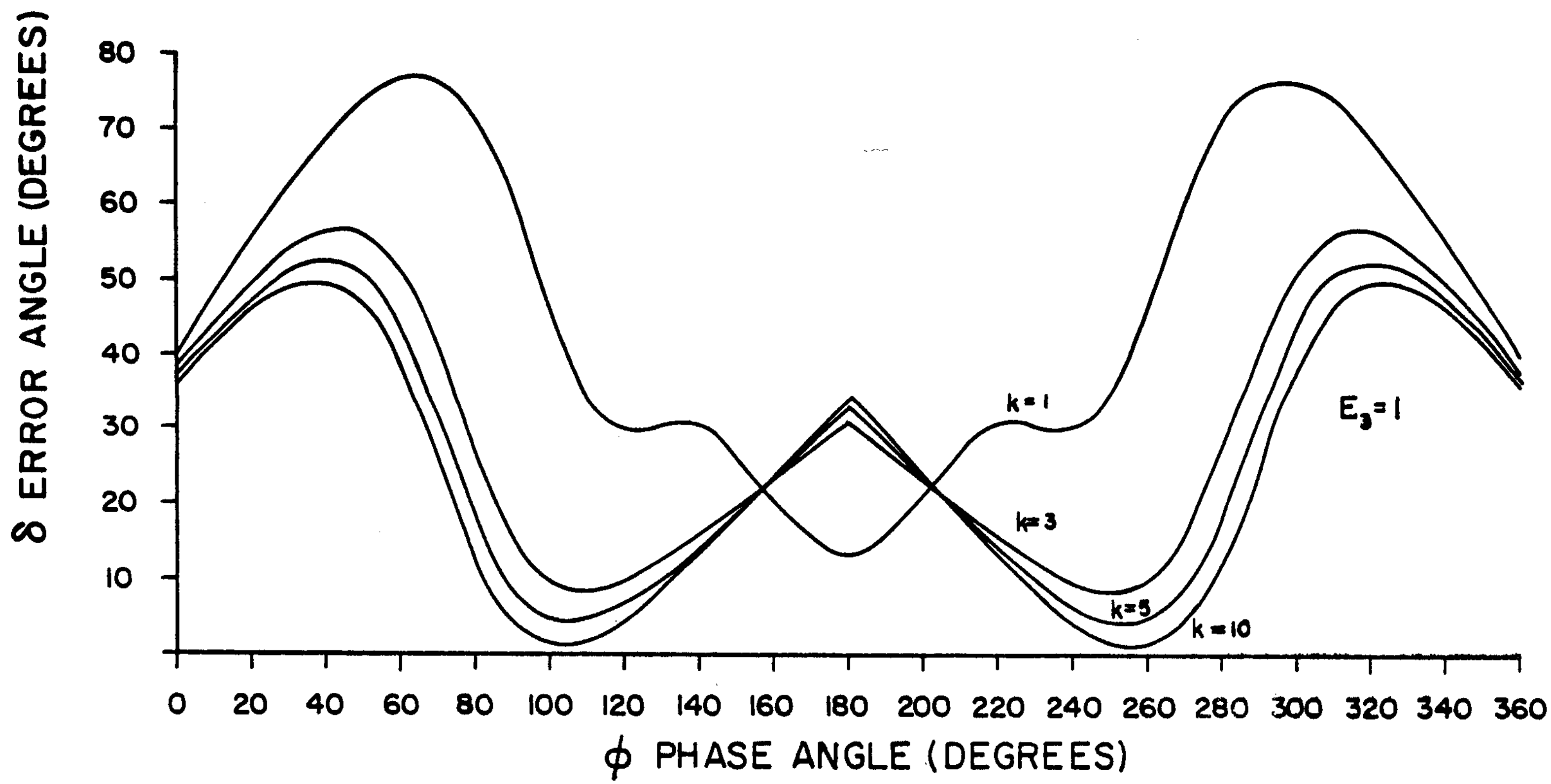
FIGURE 15



ERROR ANGLE VS PHASE ANGLE

PHASE SPLITTING NETWORK INCORRECTLY ALIGNED 30°

FIGURE 16



ERROR ANGLE VS PHASE ANGLE

PHASE SPLITTING NETWORK INCORRECTLY ALIGNED 45°

FIGURE 17

process is used to determine ϕ.

The division method does however reduce the error caused by fluctuations of the input signal, provided k is above 6 and the phase splitting network is properly aligned.

3.1.3 Error Arising from Noise Sources

Two main sources of noise are present in the phase comparator circuit, noise generated in the resistive elements and shot noise. Thermal noise arising from random motion of particles is given by the following relationship:

$$\overline{v^2} = 4kTR, \text{ per unit bandwidth}$$

Where k = Boltzmann's constant

T = Absolute Temperature

R = Resistance of the element considered.

The shot noise generated by current flowing through components may be expressed as follows:

$$\overline{i^2} = 2qI, \text{ per unit bandwidth}$$

Where q = electronic charge

I = current flowing through element considered

As the two sources of noise are uncorrelated, their powers can be added.

An analysis of the elements in the phase comparator indicates the maximum noise voltage to be expected from these sources is 3.56 microvolt developed in the diodes. For the levels of operation expected for the phase comparator, noise arising from this source is negligible.

3.1.4 Error Arising from Temperature-Sensitive Elements

The temperature-sensitive elements in the phase comparator include the diodes, the inductor, and the transformer. Temperature influences upon the inductor cause a detuning of the phase shift network resulting in deviations from quadrature.

The temperature coefficient of the inductor is 0.1% /$^{\circ}$C which results in approximately 1° shift from correct alignment for a change of 50° C.

Effects of temperature on the transformer cause a phase shift of 25° in the reference for a 50° C temperature change and is distributed over many cycles of phase change.

The most temperature-sensitive components in the phase comparator are the diodes. Two diode parameters that vary with temperature are the leakage current and the barrier voltage.

For germanium diodes the leakage current increases by a factor of approximately three for a temperature increase of 10° C, and for silicon for the same temperature change, by a factor of five Because the leakage current for silicon diodes is only of the order of 10^{-9} amps, this effect is negligible.

The other temperature-dependent diode parameter is the barrier voltage, V_b. Errors that arise from this source are minimized because the diodes are arranged so that the contact potential variations compensate one another. Consider the case of a 20% difference in the temperatures sensitivities of the two diodes over the temperature range -40° C to +80° C.

The following equation describes the barrier voltage for a PN junction,

$$V_b = \frac{KT}{q} \ln \frac{AD}{\eta_i^2} \qquad (3\text{-}3)$$

Where
K = Boltzmann's constant
T = Temperature, °K
q = Electronic charge
A = Acceptor impurity concentration
D = Donor impurity concentration
η_i = Intrinsic concentration

Assuming a typical barrier voltage, equation (3-3) may be solved for the non-temperature-sensitive product AD :

$$\ln AD = \frac{V_b q}{KT} + \ln \eta_i^2 , \qquad (3\text{-}4)$$

Where
V_b = 0.7 volt at 27° C
K = 1.38×10^{-23} joule/°K
q = 1.6×10^{-19} columb
T = 300° K
η_i^2 = $1.5 \times 10^{33} T^3 \exp(-14{,}000/T)$, for silicon.

Then

$$\ln AD = 72.17 ,$$

so that

$$V_b = KT/q\,(14{,}000/T - 3 \ln T - 3.13)$$

A 20 percent deviation between the diode parameters would give an error of - 22 mV at - 40° C and + 24 mV at 80° C. Equating this voltage to a representative angular error for the phase comparator, the phase error is 6.4°. If several cycles of phase change are available, this error can, however, be estimated.

4. Experimental Results

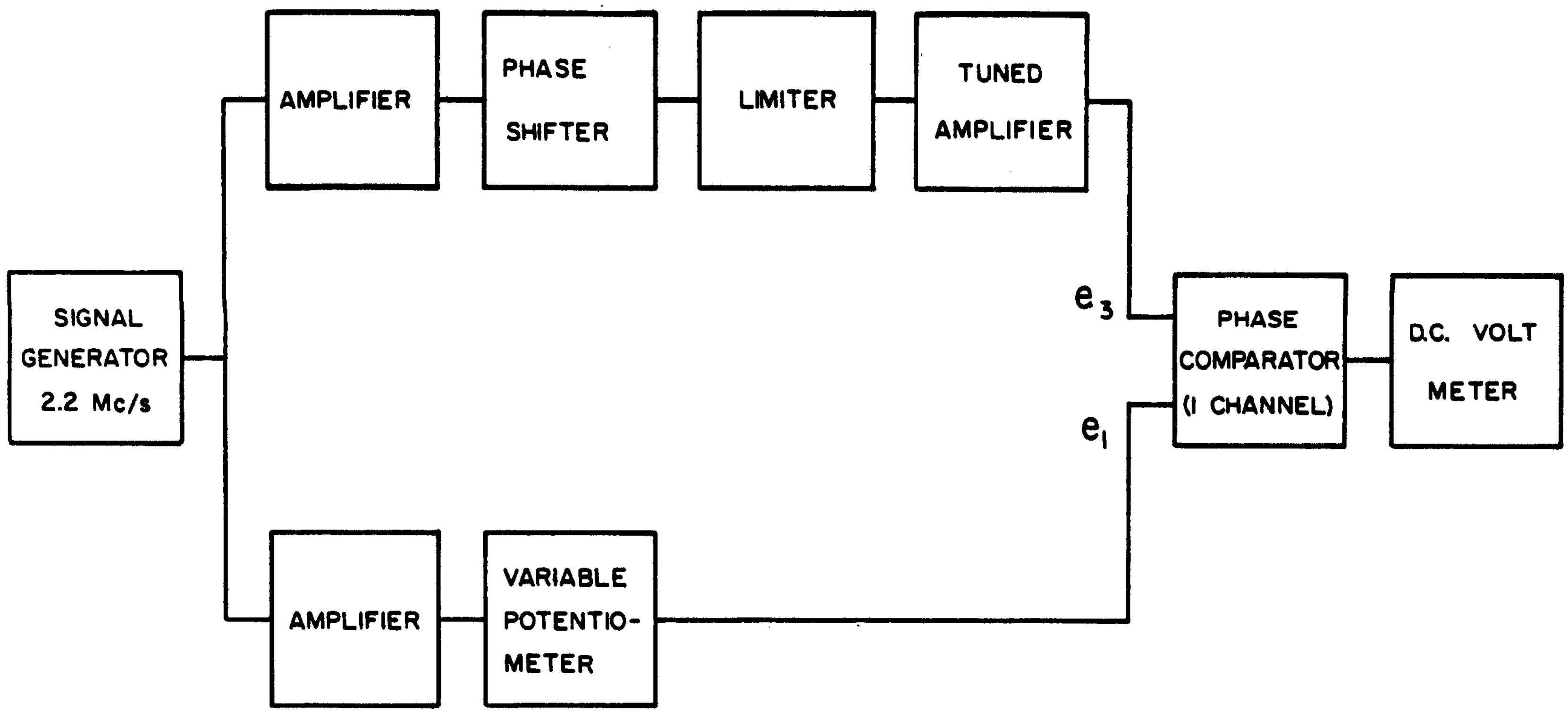
4.1 Low Frequency Measurements

The phase comparator was tested at two frequencies. A lower frequency of 2.2 mc/s was chosen because test equipment for measurements in this range was available.

The measurements at 2.2 mc/s were made on half of the phase comparator so the performance of only a single channel was measured. Two independent tests were conducted in this frequency range. A block diagram for the first measurement is shown in Figure 18. The limiter employed to reduce amplitude variations in the output of the phase shifter was developed by Brownlee (1961).

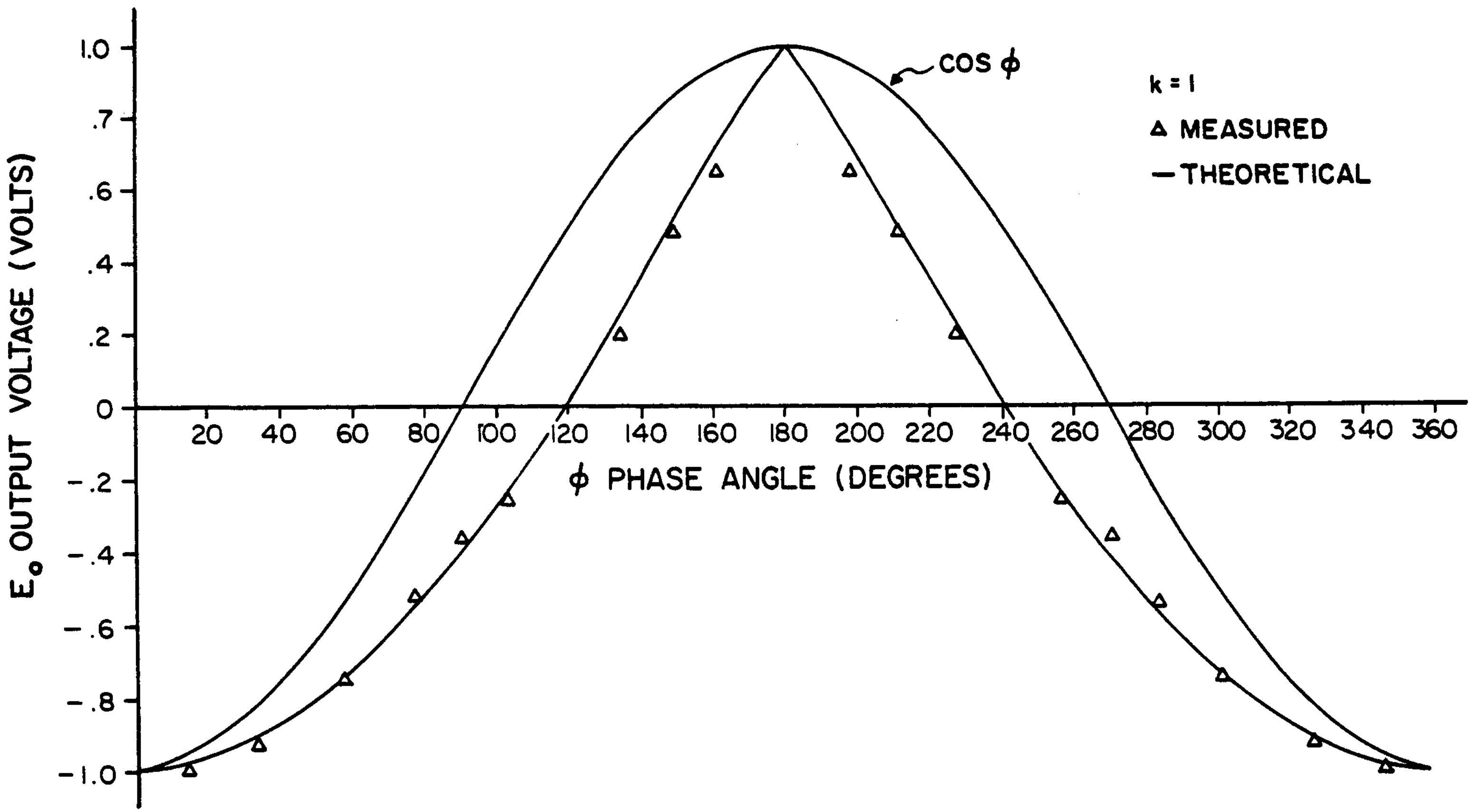
The value of k was determined by adjustment of the potentiometer. The phase between the two applied signals was determined by the position of the phase shifter and was adjusted in increments, and measured with a Tektronix 535 oscilloscope by means of Lissajous patterns. The d.c. output E_o was recorded for each phase change with a d.c. voltmeter. Tests were carried out for the following values of k: 1, 3, 5, 7, and 10. The results of these tests are shown in Figures 19, 20, 21, 22, and 23. All measured output voltages have been normalized so a direct comparison with the cosine function is possible. If d.c. balance of the diodes is not achieved a d.c. component will appear in the measured voltages and may be found from the mean voltage between the maximum and minimum output voltages.

The block diagram for the second test is shown in Figure 24.



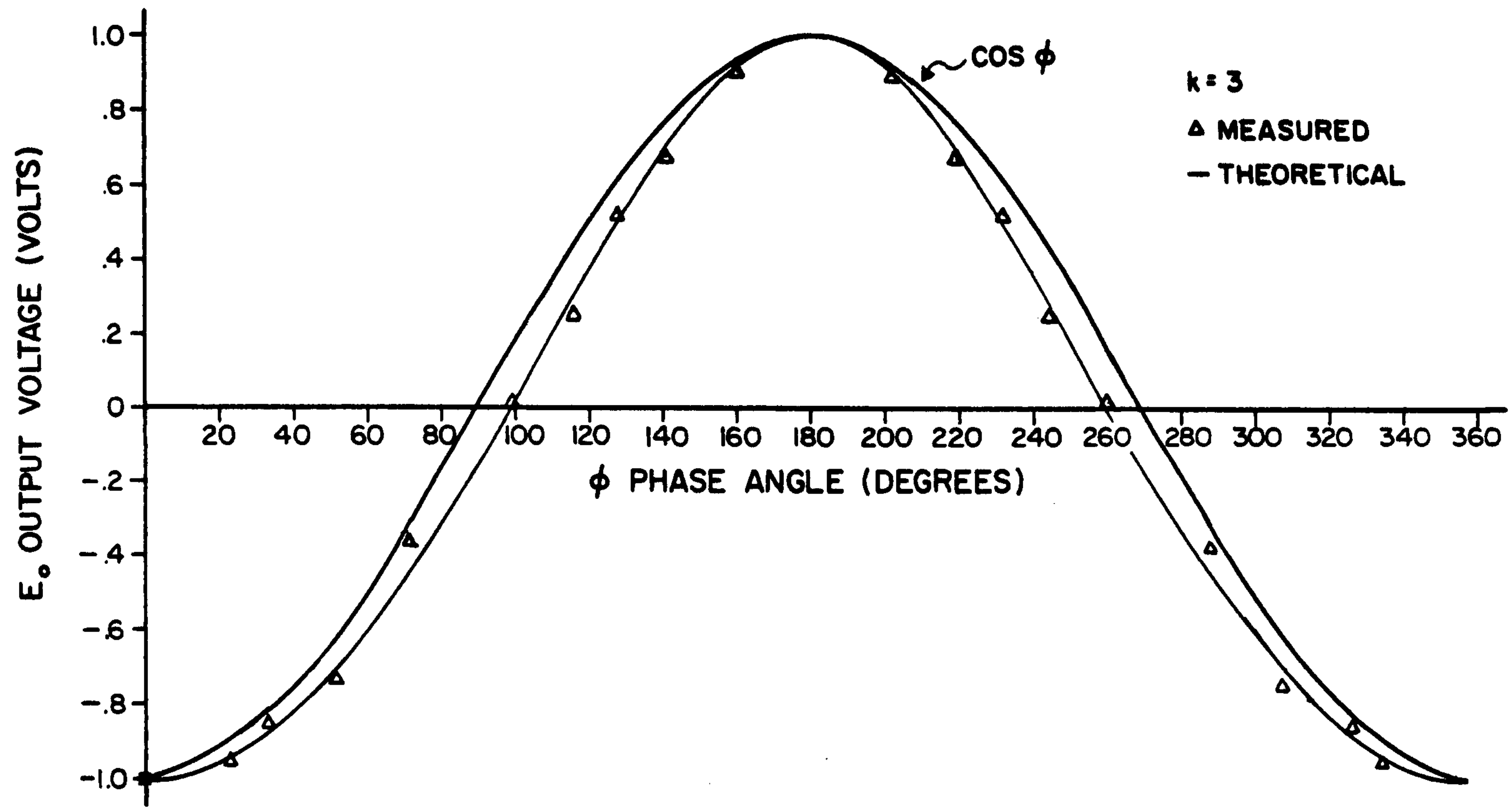
BLOCK DIAGRAM OF 2.2 MC/S TEST (FIXED PHASE MEASUREMENTS)

FIGURE 18



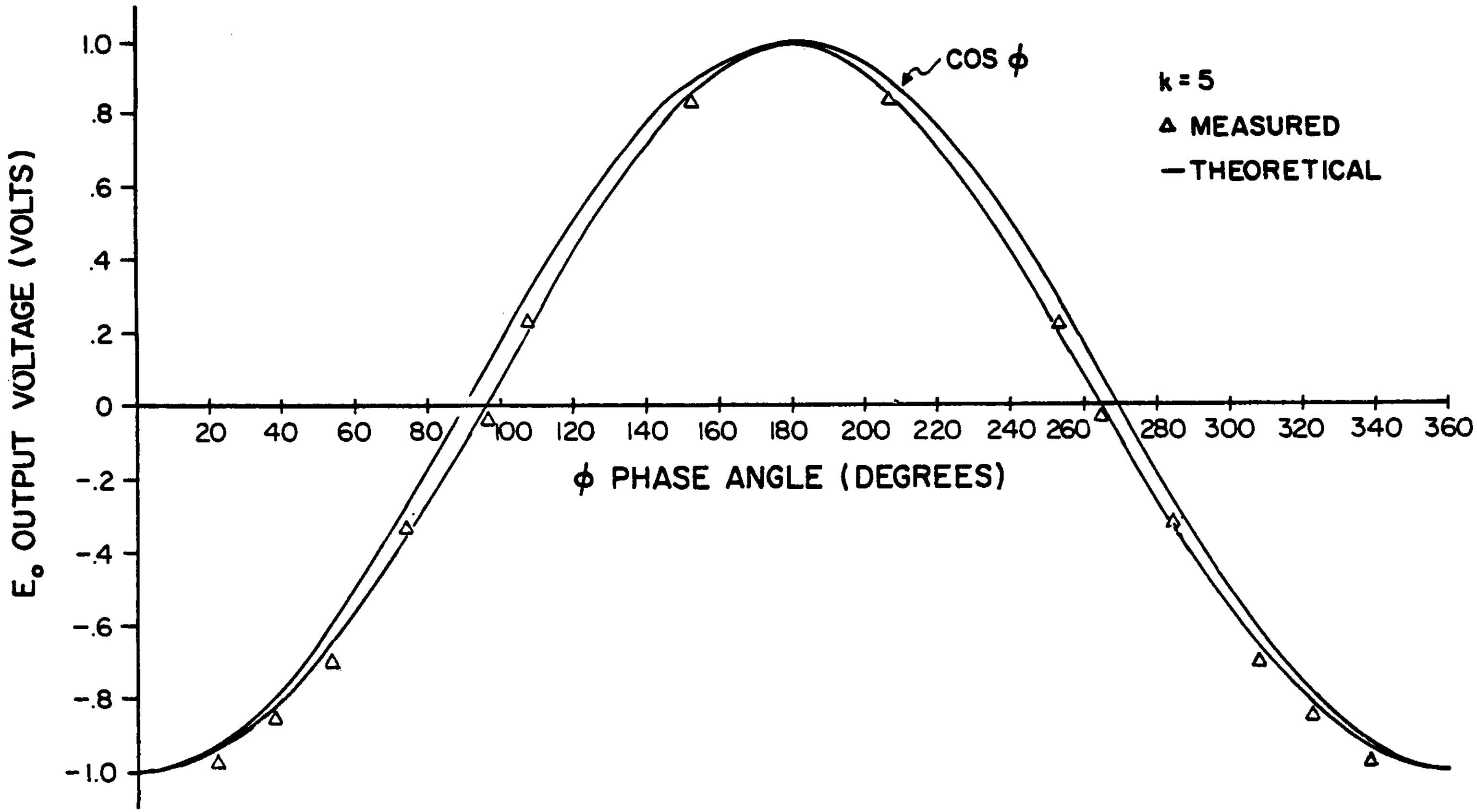
OUTPUT VOLTAGE VS PHASE ANGLE

FIGURE 19



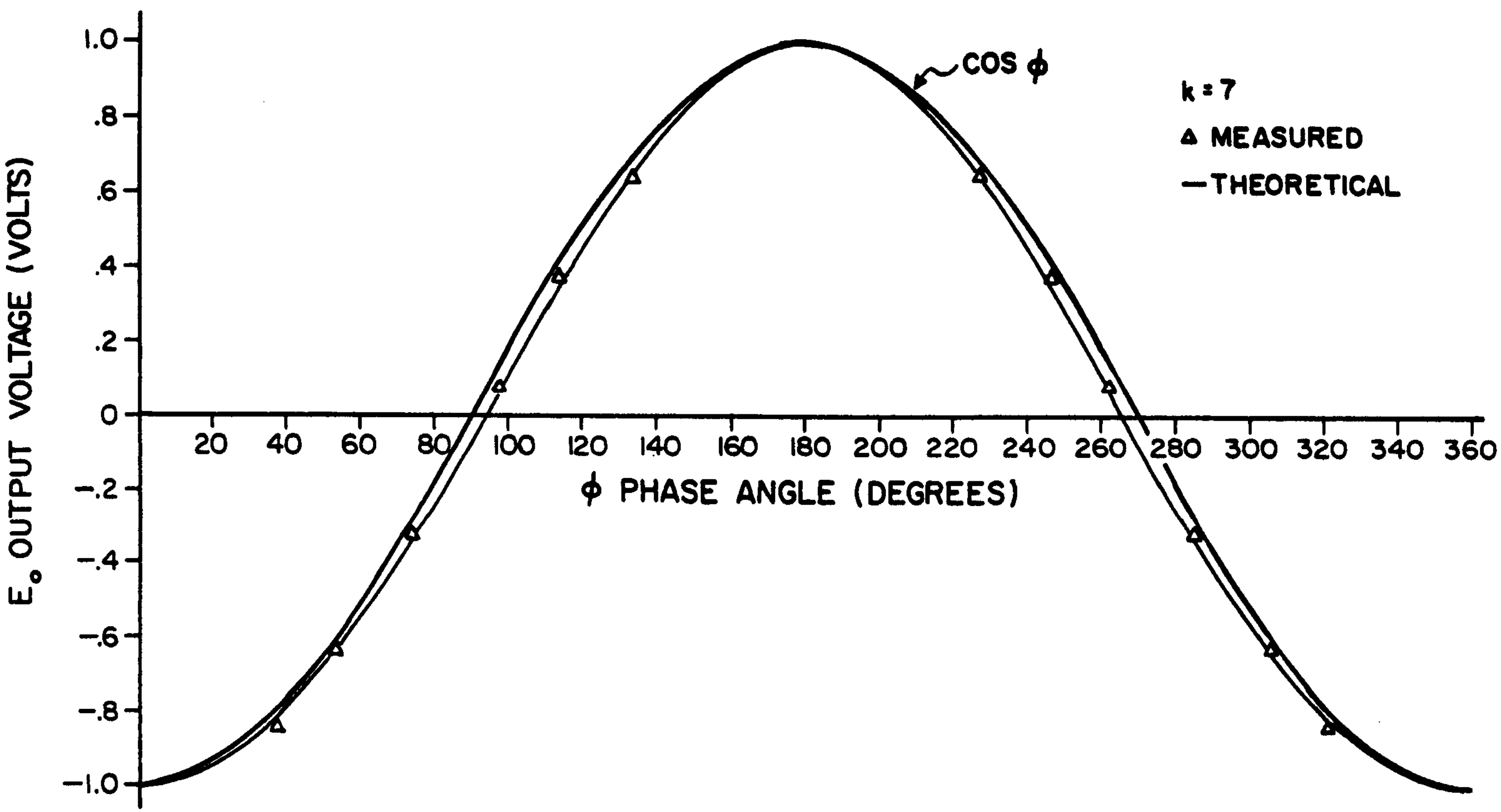
OUTPUT VOLTAGE VS PHASE ANGLE

FIGURE 20



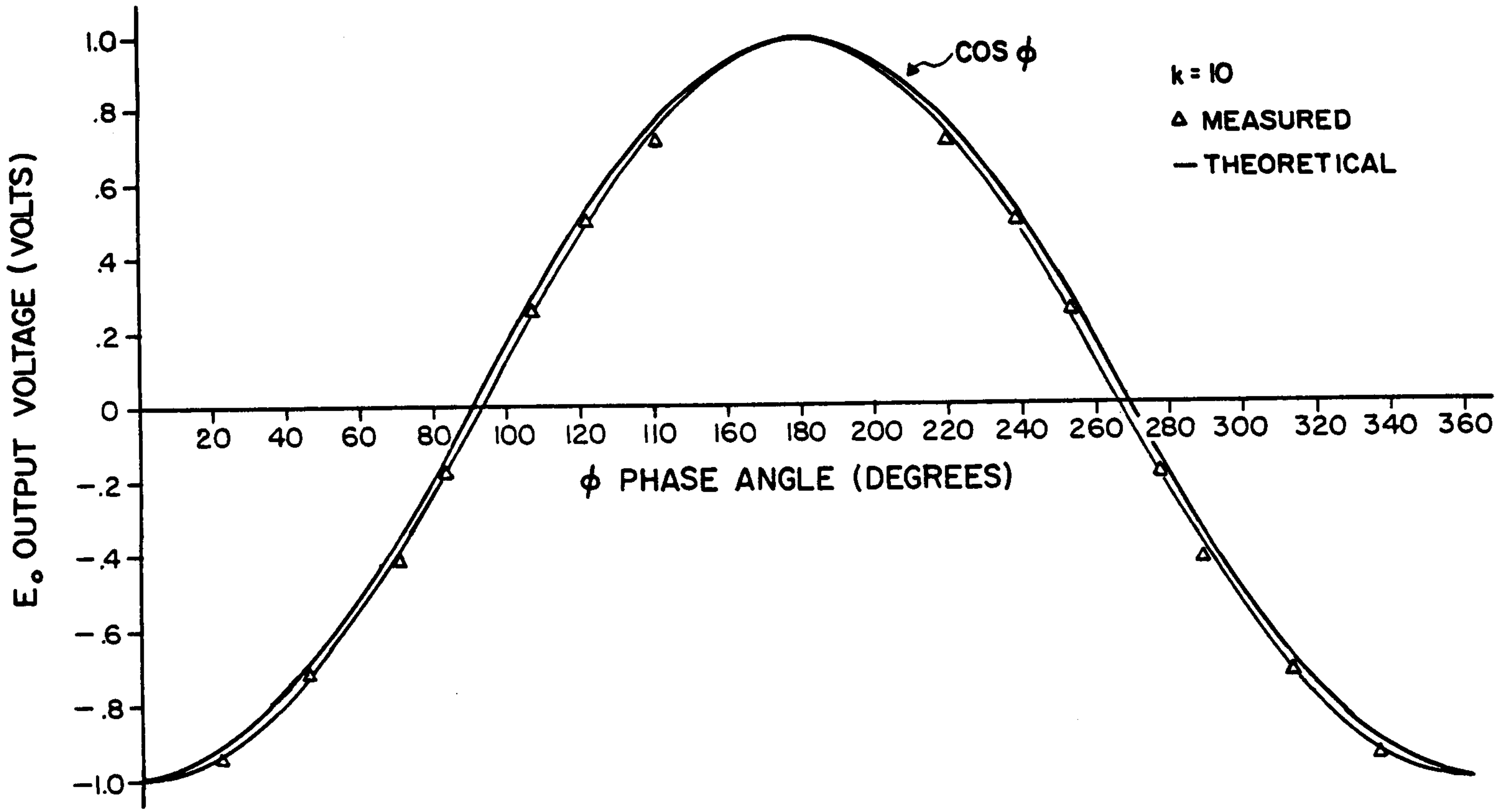
OUTPUT VOLTAGE VS PHASE ANGLE

FIGURE 21



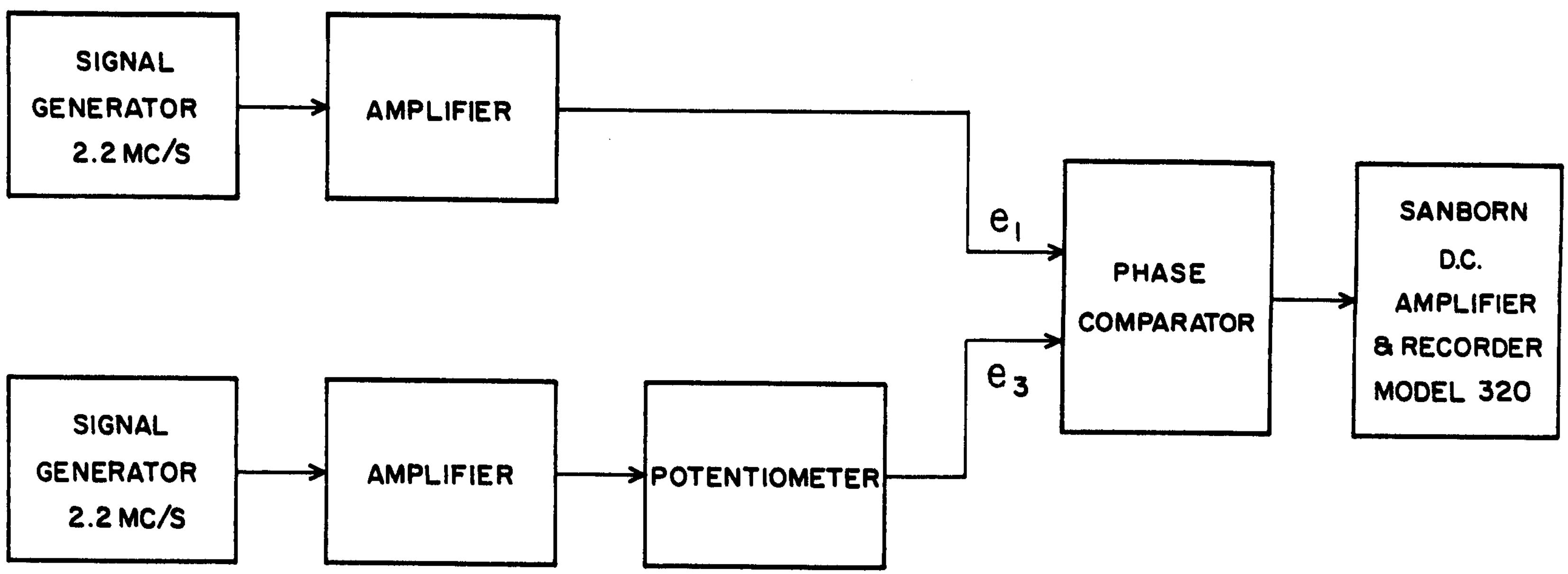
OUTPUT VOLTAGE VS PHASE ANGLE

FIGURE 22



OUTPUT VOLTAGE VS PHASE ANGLE

FIGURE 23



BLOCK DIAGRAM OF 2.2 MC/S TEST (VARIABLE PHASE)

FIGURE 24

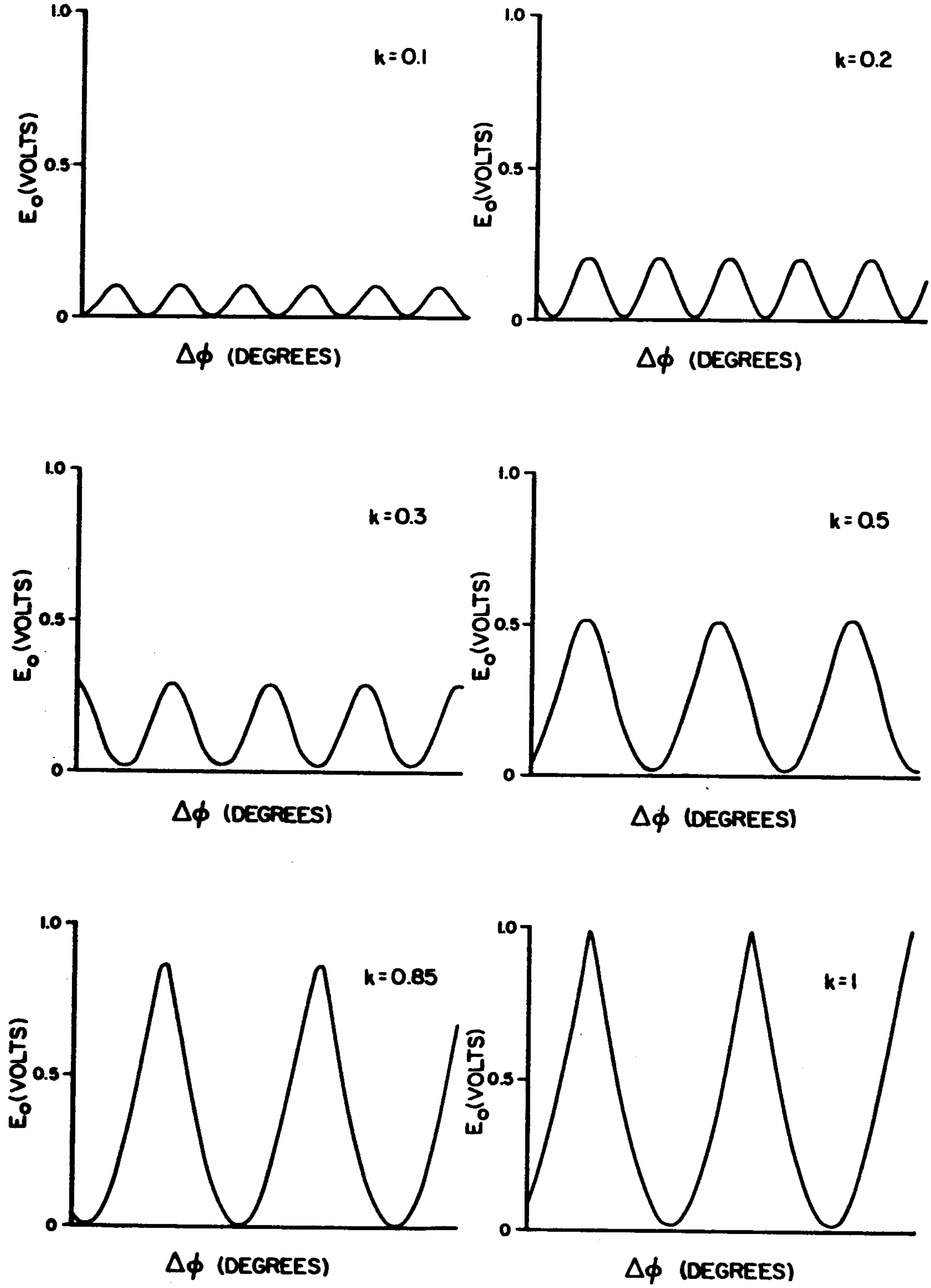
In this measurement two signal generators are adjusted to approximately the same frequency and their outputs are used to drive the phase comparator. The phase comparator output frequency is equal to the difference between the two signal generators frequencies.

The output was recorded with a Sanborn model 320 d.c. recorder. The ratio, k, was adjusted from 0.1 to 10 and the traces observed by the recorder are shown in Figures 25 and 26. The trace for k = 1 is repeated in both figures as a reference. It can be seen from these traces that when k becomes less than 1 the output is dependent on the amplitude of the smaller input voltage. As k decreases the output tends to be sinusoidal. Actually the signal now becomes the reference and the same distortion from a sinusoidal output function occurs as for the case with the roles of the two inputs reversed.

4.2 High Frequency Measurements

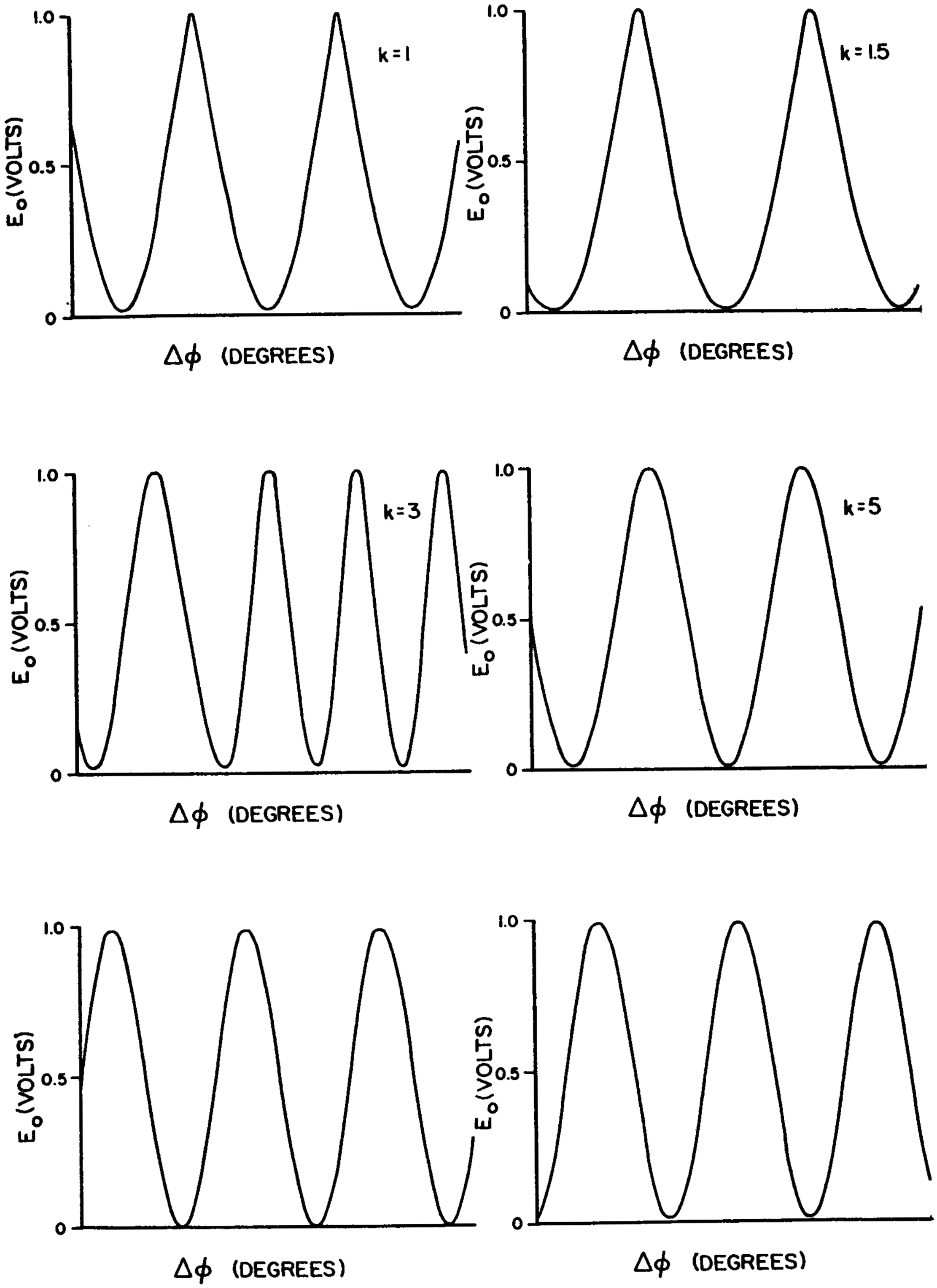
The phase comparator is designed to provide quadrature outputs at an operating frequency of 72 mc/s. A test was conducted at 72 mc/s using the complete phase comparator as shown in Figure 27.

In this test the output voltage of one channel of the phase comparator is applied to the vertical plates of an oscilloscope and the output voltage of the other channel is applied to the horizontal plates. The ratio, k, was varied as before and pictures of the resultant trace were taken with a polaroid oscilloscope camera. These pictures are shown in Figures 28, 39, and 30. When the ratio of k was 10, the input signal was reduced in order not to exceed the diode breakdown



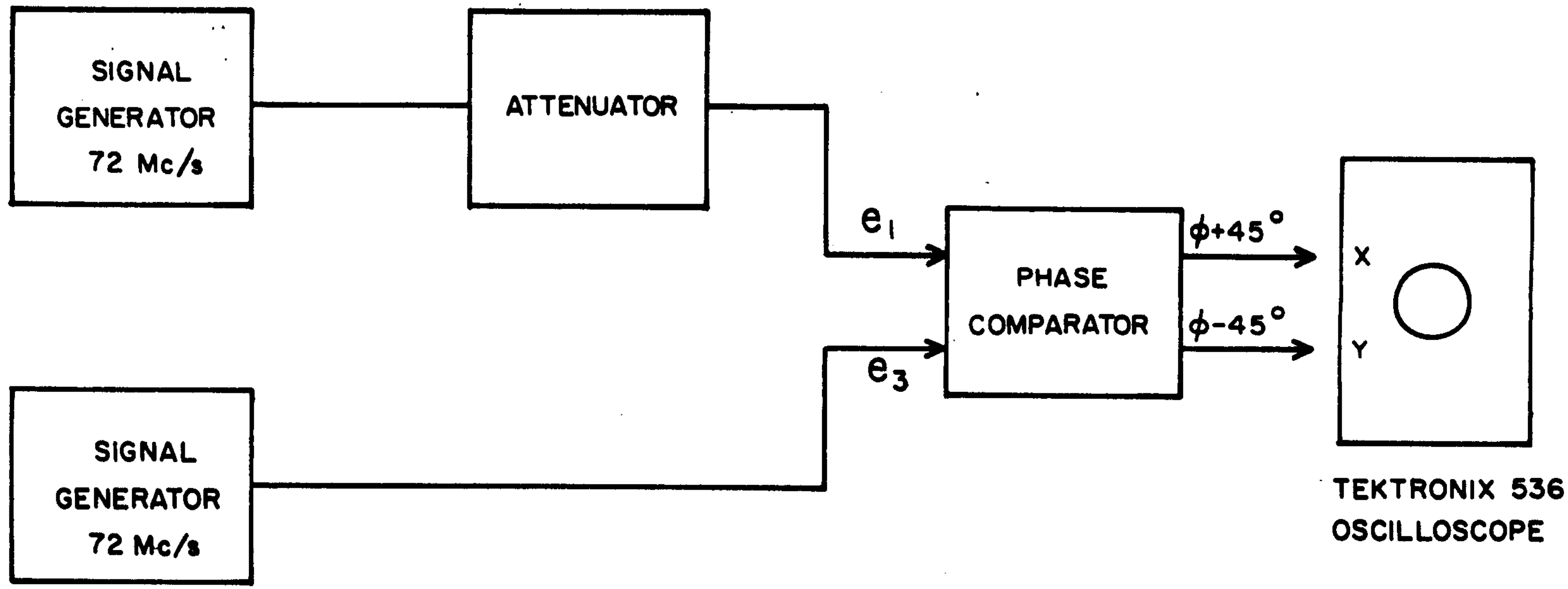
OUTPUT VOLTAGE VS. PHASE ANGLE k<1

FIGURE 25



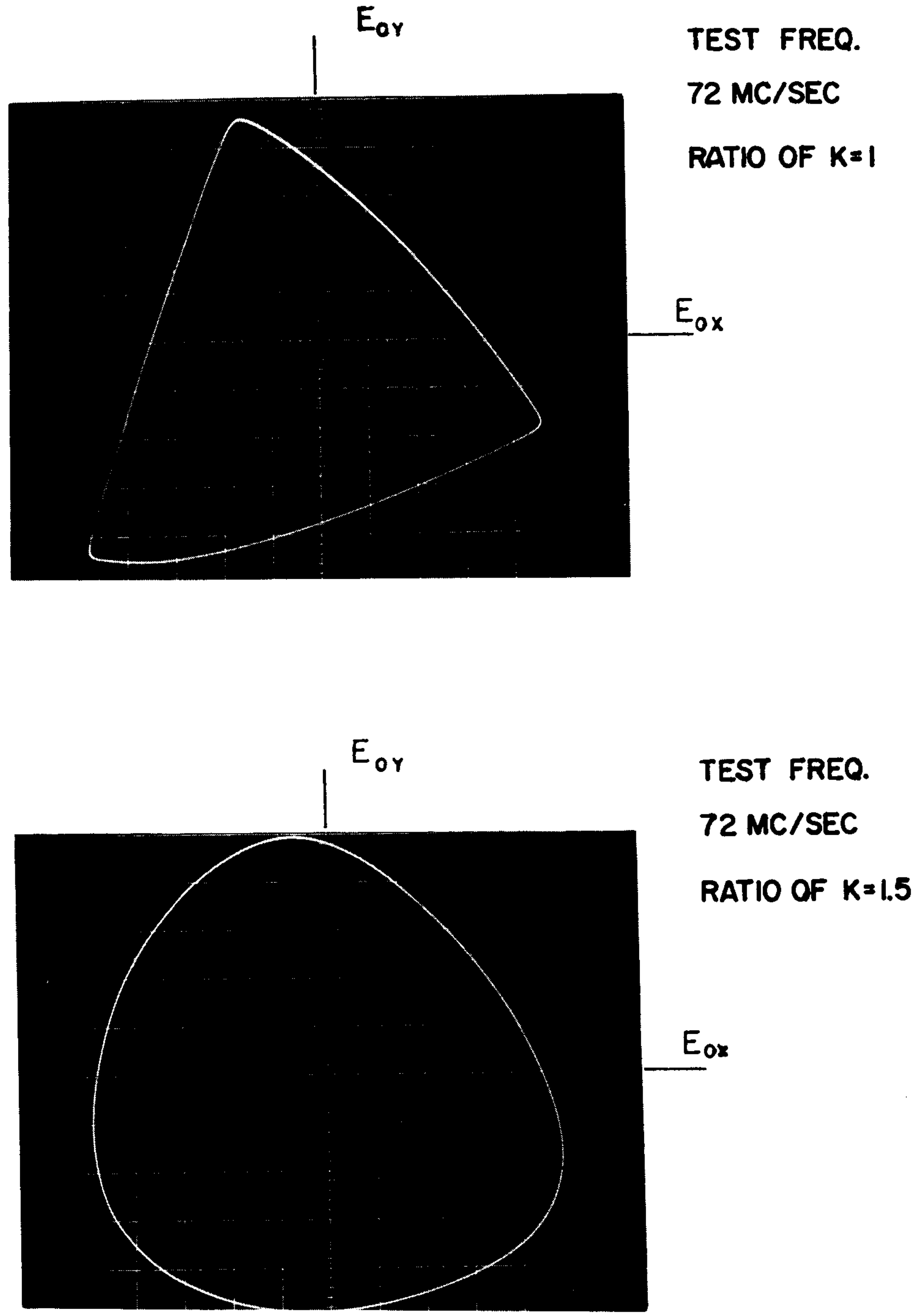
OUTPUT VOLTAGE VS PHASE ANGLE k<1

FIGURE 26



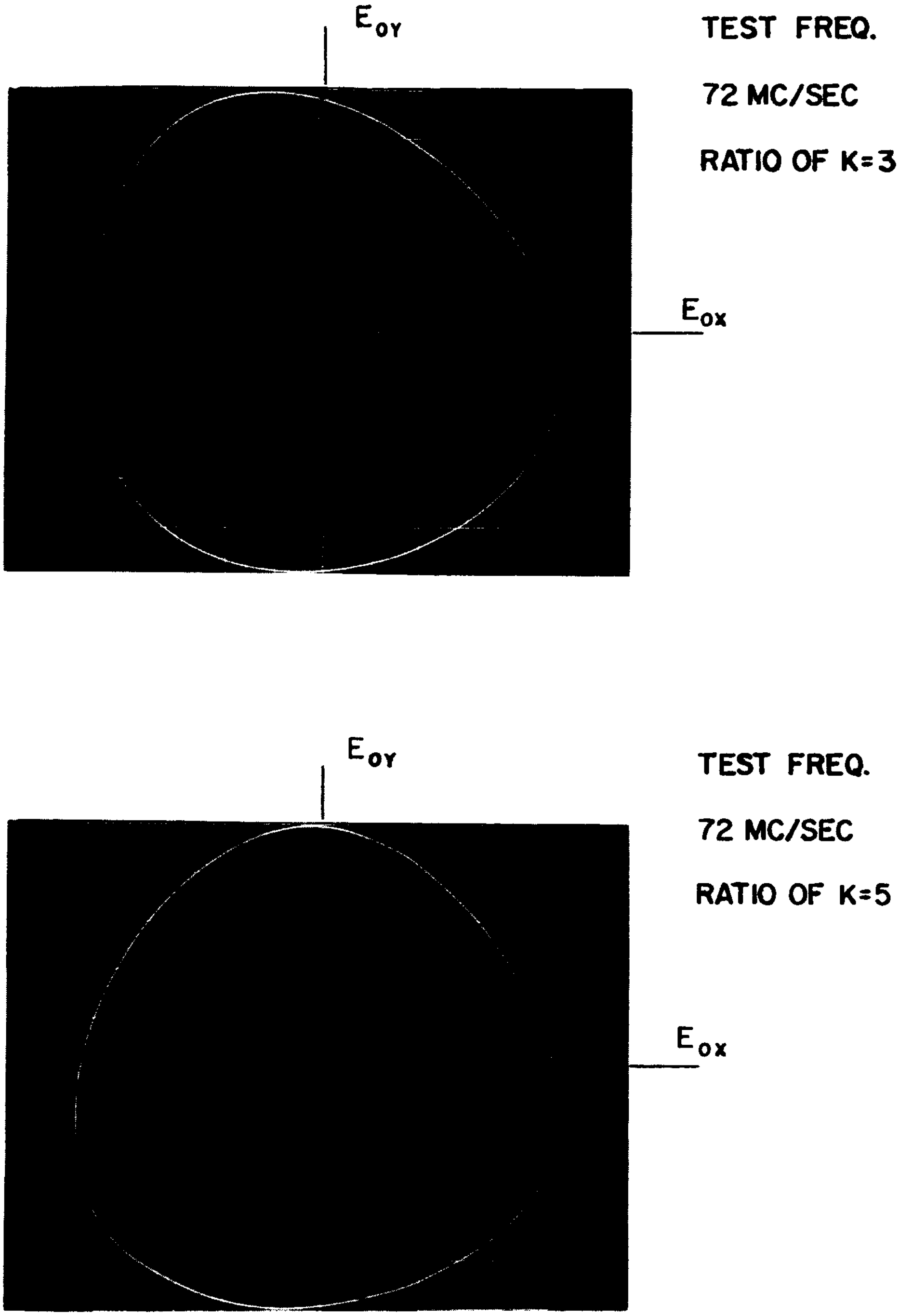
BLOCK DIAGRAM OF 72 MC/S TEST

FIGURE 27



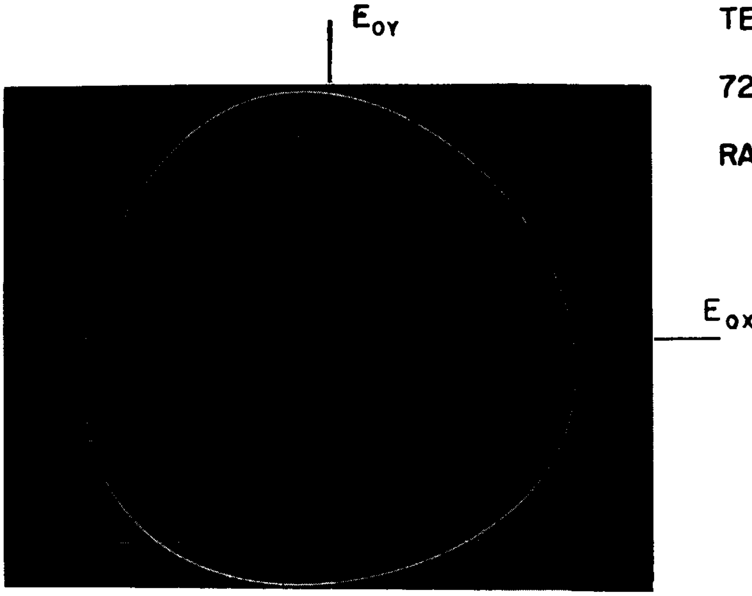
QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR

FIGURE 28



QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR

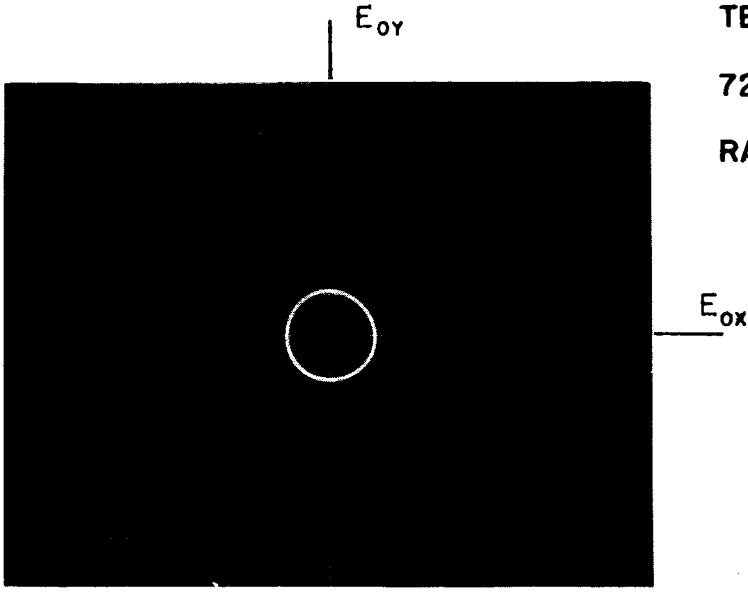
FIGURE 29



TEST FREQ.

72 MC/SEC

RATIO OF K = 7



TEST FREQ.

72 MC/SEC

RATIO OF K = 10

QUADRATURE OUTPUT CHARACTERISTICS OF PHASE COMPARATOR

FIGURE 30

potential. This resulted in a reduction in the radius of the trace on the scope.

In all of the tests performed on the phase comparator the results agreed with theoretical predictions within about 2 percent.

5. Summary and Conclusions

Methods of comparing phase at 72 mc/s were examined to find the relative advantages of each for a rocket borne application. A phase comparator capable of measuring phase to within $\pm 5^{\circ}$, with the additional requirements of light weight, small volume, and low power consumption was required. Phase comparators in general were investigated and it was found that to measure phase at 72 mc/s, comparators which use vector rectifiers were the most suitable for rocket instrumentation. A phase comparator employing vector rectifiers and requiring fewer components than units previously used for phase comparison was designed by Fisher of Spacecraft, Inc. This unit was analyzed and tested.

It was found that the output voltages from this phase comparator are proportional to the cosine of the phase angle between the two inputs, the efficiency of the diode vector rectifiers, and the amplitude of the smaller input voltage.

The potential sources of error were investigated. It was found that if the ratio of the input voltages k is maintained greater than 6, a sinusoidal output may be assumed to obtain the desired phase measurement to within $\pm 5^{\circ}$. For values of $k < 6$, error from this source may be minimized by using the appropriate calibration curve for the phase angle measurement. Measurements of phase angles can be made to within about 2% by this method.

A method commonly used in the analysis of data obtained from a quadrature output phase comparator is to divide the two output voltages to determine a measure of the tangent of the phase angle. The phase angle obtained is independent of efficiency and input voltage fluctuations.

The analysis showed that this method could only be employed for values of $k > 6$ to obtain the required accuracy because of the non-symmetric relationship of the output for smaller ratios. In addition, the phase splitting network must be correctly aligned to assure that a quadrature relationship exists between the two channels. Considerable error may be introduced as the result of the division process if the phase difference between the two reference voltages is not 90°.

Noise is not considered to be a problem since the diode vector rectifiers operate at relatively high levels.

The diode configuration is such that temperature effects are self-compensating. A 20% difference in the diode characteristics was considered and it was predicted that this difference would result in an overall error of approximately 6.4° during the time of a proposed rocket experiment.

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